Projection of Private Values in Auctions

Tristan Gagnon-Bartsch	Marco Pagnozzi	Antonio Rosato
Harvard	Naples "Federico II"	UTS

Very Preliminary

Motivation

- Evidence from social psychology suggests that people mispredict others' preferences in a systematic way
- **False-consensus Effect** or **Taste Projection**: people think others' tastes are more similar to their own (than they actually are)

  e.g., those with taste for particular varieties of products, art, or sports tend to overestimate how many share that taste (Ross *et al.*, 1977)

  - Such misperceptions arise in many other relevant domains: political preferences (Delavande and Manski, 2012); risk preferences (Faro and Rottenstreich, 2006); preferences for income redistribution (Cruces *et al.*, 2013); paternalistic interventions (Ambuehl *et al.*, 2019); jokes (Kleinberg *et al.*, 2018)

- While there is a large empirical literature documenting such a bias, relatively little research studies its economic implications

Motivation

- Evidence from social psychology suggests that people mispredict others' preferences in a systematic way
- **False-consensus Effect** or **Taste Projection**: people think others' tastes are more similar to their own (than they actually are)

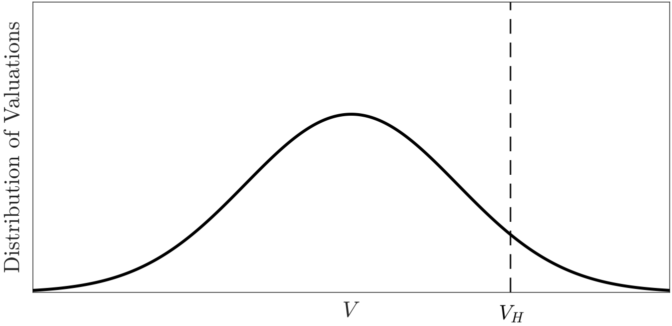
  e.g., those with taste for particular varieties of products, art, or sports tend to overestimate how many share that taste (Ross *et al.*, 1977)

  - Such misperceptions arise in many other relevant domains: political preferences (Delavande and Manski, 2012); risk preferences (Faro and Rottenstreich, 2006); preferences for income redistribution (Cruces *et al.*, 2013); paternalistic interventions (Ambuehl *et al.*, 2019); jokes (Kleinberg *et al.*, 2018)
- While there is a large empirical literature documenting such a bias, relatively little research studies its economic implications

Taste Projection in Auctions: Example (FPA)

- Potential home buyers submitting bids to a seller
  - Potential buyers differ in how much they value various features
    - Design, view, garden, neighborhood, etc.
  - Goal: guess value of highest-value opponent and slightly out bid
  - Problem: strongly value house ⇒ exaggerate extent that others do too

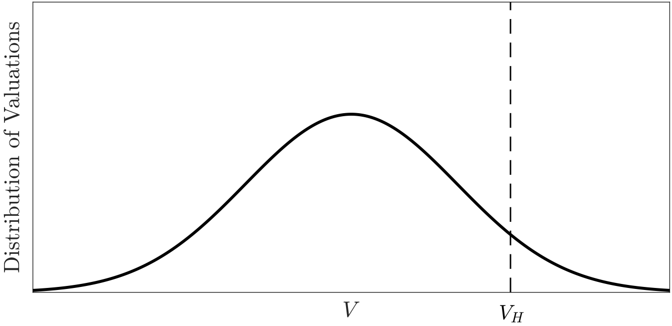
⇒ Projection causes bidders to exaggerate competition and overbid



Taste Projection in Auctions: Example (FPA)

- Potential home buyers submitting bids to a seller
  - Potential buyers differ in how much they value various features
    - Design, view, garden, neighborhood, etc.
  - Goal: guess value of highest-value opponent and slightly out bid
  - Problem: strongly value house ⇒ exaggerate extent that others do too

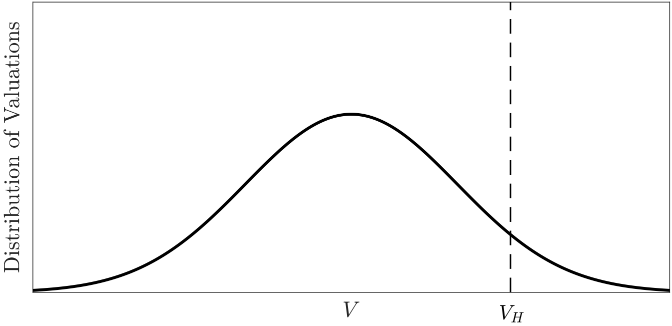
⇒ Projection causes bidders to exaggerate competition and overbid



Taste Projection in Auctions: Example (FPA)

- Potential home buyers submitting bids to a seller
  - Potential buyers differ in how much they value various features
    - Design, view, garden, neighborhood, etc.
  - Goal: guess value of highest-value opponent and slightly out bid
  - Problem: strongly value house ⇒ exaggerate extent that others do too

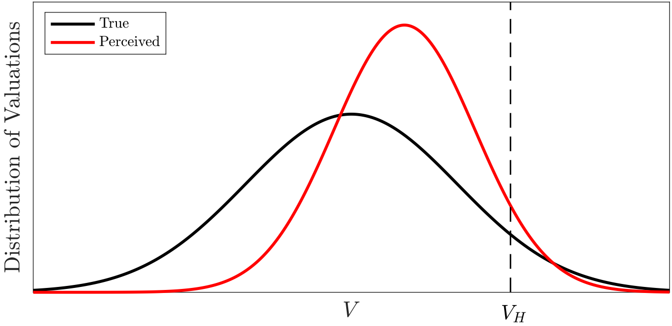
⇒ Projection causes bidders to exaggerate competition and overbid



Taste Projection in Auctions: Example (FPA)

- Potential home buyers submitting bids to a seller
  - Potential buyers differ in how much they value various features
    - Design, view, garden, neighborhood, etc.
  - Goal: guess value of highest-value opponent and slightly out bid
  - Problem: strongly value house $\Rightarrow$ exaggerate extent that others do too

$\Rightarrow$ Projection causes bidders to exaggerate competition and overbid



Taste Projection in Auctions: Example (English)

- Suppose bidders also have private information about a commonly-valued quality of the house
  - Quality of the construction, risk of flood/landslide
- A potential buyer walks away from current asking price
  - Question: What do remaining buyers infer?
  - Motive depends on two components: private taste and information
- Projectors exaggerate extent to which they know others' tastes

  $\Rightarrow$ interpretation of information shaped by observer's taste

    - Low private value: not surprised that potential buyer walked away
    - High private value: quite surprised $\Rightarrow$ pessimistic about common value

Taste Projection in Auctions: Example (English)

- Suppose bidders also have private information about a commonly-valued quality of the house
  - Quality of the construction, risk of flood/landslide
- A potential buyer walks away from current asking price
  - Question: What do remaining buyers infer?
  - Motive depends on two components: private taste and information
- Projectors exaggerate extent to which they know others' tastes

  $\Rightarrow$ interpretation of information shaped by observer's taste
  - Low private value: not surprised that potential buyer walked away
  - High private value: quite surprised $\Rightarrow$ pessimistic about common value

Taste Projection in Auctions: Example (English)

- Suppose bidders also have private information about a commonly-valued quality of the house
  - Quality of the construction, risk of flood/landslide
- A potential buyer walks away from current asking price
  - Question: What do remaining buyers infer?
  - Motive depends on two components: private taste and information
- Projectors exaggerate extent to which they know others' tastes

  $\Rightarrow$ interpretation of information shaped by observer's taste

    - Low private value: not surprised that potential buyer walked away
    - High private value: quite surprised $\Rightarrow$ pessimistic about common value

Taste Projection in Auctions: Example (English)

- Suppose bidders also have private information about a commonly-valued quality of the house
  - Quality of the construction, risk of flood/landslide
- A potential buyer walks away from current asking price
  - Question: What do remaining buyers infer?
  - Motive depends on two components: private taste and information
- Projectors exaggerate extent to which they know others' tastes

  $\Rightarrow$ interpretation of information shaped by observer's taste

    - Low private value: not surprised that potential buyer walked away
    - High private value: quite surprised $\Rightarrow$ pessimistic about common value

Taste Projection in Auctions

- We explore how taste projection affects bidding strategies, efficiency, and revenue across different auction environments and formats
- With projection, bidders with different private values perceive the distribution of values differently
- Projection induces perceived distribution of values that's less dispersed

  ⇒ **Competition effect**
- Projection introduces an additional effect if the good has a common-value element about which bidders have private info:
  - Bidders use opponents' strategies to obtain information about the common value
  - As projectors have wrong beliefs about others' tastes, they systematically draw biased inferences from others' strategies

  ⇒ **Misinference effect**

Taste Projection in Auctions

- We explore how taste projection affects bidding strategies, efficiency, and revenue across different auction environments and formats
- With projection, bidders with different private values perceive the distribution of values differently
- Projection induces perceived distribution of values that's less dispersed

  ⇒ **Competition effect**

- Projection introduces an additional effect if the good has a common-value element about which bidders have private info:
  - Bidders use opponents' strategies to obtain information about the common value
  - As projectors have wrong beliefs about others' tastes, they systematically draw biased inferences from others' strategies

  ⇒ **Misinference effect**

Taste Projection in Auctions

- We explore how taste projection affects bidding strategies, efficiency, and revenue across different auction environments and formats
- With projection, bidders with different private values perceive the distribution of values differently
- Projection induces perceived distribution of values that's less dispersed

  $\Rightarrow$ **Competition effect**
- Projection introduces an additional effect if the good has a common-value element about which bidders have private info:
  - Bidders use opponents' strategies to obtain information about the common value
  - As projectors have wrong beliefs about others' tastes, they systematically draw biased inferences from others' strategies

  $\Rightarrow$ **Misinference effect**

Taste Projection in Auctions

- We explore how taste projection affects bidding strategies, efficiency, and revenue across different auction environments and formats
- With projection, bidders with different private values perceive the distribution of values differently
- Projection induces perceived distribution of values that's less dispersed

  $\Rightarrow$ **Competition effect**

- Projection introduces an additional effect if the good has a common-value element about which bidders have private info:
  - Bidders use opponents' strategies to obtain information about the common value
  - As projectors have wrong beliefs about others' tastes, they systematically draw biased inferences from others' strategies

  $\Rightarrow$ **Misinference effect**

Preview of the results

- First-price auctions yield higher revenue than second-price auctions

  ⇒ Revenue equivalence no longer holds

- With an uncertain common-value element, taste projection:
  - Lowers efficiency in second-price auctions

  ⇒ First-price auctions more efficient than second-price ones

- With taste projection, optimal reserve price
  - is lower than rational in first-price auctions
  - might vary with the number of bidders in first-price auctions

  ⇒ different reserve price in first-price auctions than second-price ones

Preview of the results

- First-price auctions yield higher revenue than second-price auctions

  ⇒ Revenue equivalence no longer holds

- With an uncertain common-value element, taste projection:

  - Lowers efficiency in second-price auctions

  ⇒ First-price auctions more efficient than second-price ones

- With taste projection, optimal reserve price

  - is lower than rational in first-price auctions

  - might vary with the number of bidders in first-price auctions

  ⇒ different reserve price in first-price auctions than second-price ones

Preview of the results

- First-price auctions yield higher revenue than second-price auctions

  ⇒ Revenue equivalence no longer holds

- With an uncertain common-value element, taste projection:

  - Lowers efficiency in second-price auctions

  ⇒ First-price auctions more efficient than second-price ones

- With taste projection, optimal reserve price

  - is lower than rational in first-price auctions
  - might vary with the number of bidders in first-price auctions

  ⇒ different reserve price in first-price auctions than second-price ones

Related Literature

1. *Inter*personal Projection Bias and False-Consensus Effect
    - Early psych literature finds strong correlation between own preferences and prediction of others' (Ross *et al.*, 1977; Mullen *et al.*, 1985; Krueger & Clement, 1994)
    - Appears inconsistent with Bayesian rationality (Loewenstein & VanBoven, 2003; Engelmann & Strobel, 2012)
    - Some theoretical work on causes and implications: Gagnon-Bartsch (2016); Bohren & Hauser (2018); Breitmoser (2019); Frick, Iijima and Ishii (2019a, 2019b)
2. *Intra*personal Projection Bias, Cursedness and Information Projection
    - Loewenstein, O'Donoghue and Rabin (2003), Eyster & Rabin (2005), Madarasz (2012, 2016)
3. Auctions with Private and Common Values
    - Klemperer (1998), Compte & Jehiel (2002), Goeree & Offerman (2002, 2003), Pagnozzi (2007), Hernando-Veciana (2009), Larson (2009)

Outline

Model

Private Values

Private and Common Values

Extensions

Conclusions

Environment

- $N \geq 2$ bidders; bidder i's valuation is

$$V_i = t_i + \gamma \sum_{j=1}^{N} \theta_j$$

  - t_i is private value or "taste"
  - θ_i is (private) signal about the common value $\sum_{j=1}^{N} \theta_j$
  - $\gamma \geq 0$ measures the relative weight put on the common value
- Private values $t_i \overset{\text{iid}}{\sim} F$ with log-concave density f on $[\underline{t}, \overline{t}] \subseteq \mathbb{R}$
- Signals $\theta_i \overset{\text{iid}}{\sim} G$ with log-concave density g on $\left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}$
- Signals about common value are independent of private values
- Analyze first-price (and Dutch) vs. second-price (and English)
- Focus on symmetric strategies (for reasons apparent later)

Environment

- $N \geq 2$ bidders; bidder i's valuation is

$$V_i = t_i + \gamma \sum_{j=1}^{N} \theta_j$$

  - t_i is private value or "taste"
  - θ_i is (private) signal about the common value $\sum_{j=1}^{N} \theta_j$
  - $\gamma \geq 0$ measures the relative weight put on the common value
- Private values $t_i \overset{\text{iid}}{\sim} F$ with log-concave density f on $[\underline{t}, \overline{t}] \subseteq \mathbb{R}$
- Signals $\theta_i \overset{\text{iid}}{\sim} G$ with log-concave density g on $\left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}$
- Signals about common value are independent of private values
- Analyze first-price (and Dutch) vs. second-price (and English)
- Focus on symmetric strategies (for reasons apparent later)

Environment

- $N \geq 2$ bidders; bidder i's valuation is

$$V_i = t_i + \gamma \sum_{j=1}^{N} \theta_j$$

  - t_i is private value or "taste"
  - θ_i is (private) signal about the common value $\sum_{j=1}^{N} \theta_j$
  - $\gamma \geq 0$ measures the relative weight put on the common value
- Private values $t_i \overset{\text{iid}}{\sim} F$ with log-concave density f on $[\underline{t}, \overline{t}] \subseteq \mathbb{R}$
- Signals $\theta_i \overset{\text{iid}}{\sim} G$ with log-concave density g on $\left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}$
- Signals about common value are independent of private values
- Analyze first-price (and Dutch) vs. second-price (and English)
- Focus on symmetric strategies (for reasons apparent later)

Taste Projection: Parametric Model

- Bidder i believes j's private value is $\hat{t}_j(t_i) = \alpha t_i + (1-\alpha) t_j$
  - Similar to Loewenstein *et al.* (2003) model of *intra*personal projection
  - $\alpha \in [0,1]$ parameterizes extent of projection
  - $\alpha = 0 \Rightarrow$ rational benchmark, $\alpha = 1 \Rightarrow$ "full" projection

$\Rightarrow$ Player i's model: tastes are i.i.d. realizations of random variable

$$\hat{t}(t_i) = \alpha t_i + (1-\alpha) t \quad \text{where } t \sim F$$

- **Extension to correlated private values:** Bidder i with private value t_i believes j's taste is realization of random variable

$$\hat{t}_j(t_i) = \alpha t_i + (1-\alpha)\tilde{t}_j(t_i)$$

where $\tilde{t}_j(t_i)$ has Bayesian posterior distribution given t_i

Taste Projection: Parametric Model

- Bidder i believes j's private value is $\hat{t}_j(t_i) = \alpha t_i + (1-\alpha)t_j$
  - Similar to Loewenstein *et al.* (2003) model of *intra*personal projection
  - $\alpha \in [0,1]$ parameterizes extent of projection
  - $\alpha = 0 \Rightarrow$ rational benchmark, $\alpha = 1 \Rightarrow$ "full" projection

$\Rightarrow$ Player i's model: tastes are i.i.d. realizations of random variable

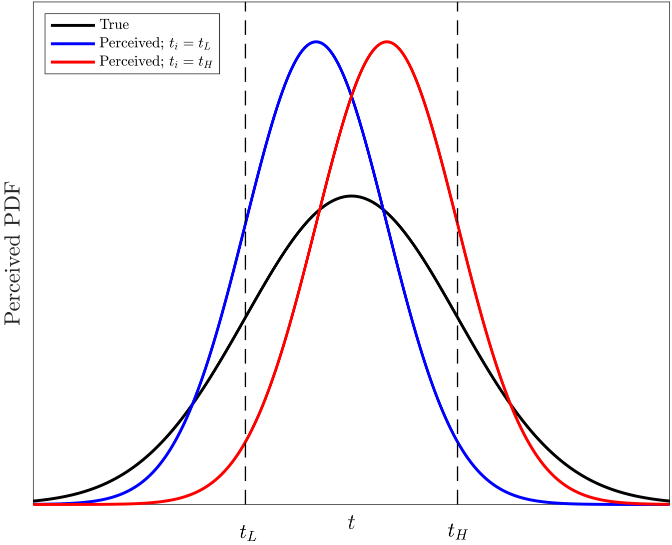
$$\hat{t}(t_i) = \alpha t_i + (1-\alpha)t \quad \text{where } t \sim F$$

- **Extension to correlated private values:** Bidder i with private value t_i believes j's taste is realization of random variable

$$\hat{t}_j(t_i) = \alpha t_i + (1-\alpha)\tilde{t}_j(t_i)$$

where $\tilde{t}_j(t_i)$ has Bayesian posterior distribution given t_i

Taste Projection: Parametric Model



Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$

    1. Projected distribution is taste-dependent
    2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
    3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2\,\mathrm{Var}(t)$
    4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \;\Leftrightarrow\; t_i \geq t_j$
    5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$

  1. Projected distribution is taste-dependent
  2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
  3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2 \mathrm{Var}(t)$
  4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \;\Leftrightarrow\; t_i \geq t_j$
  5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$

  1. Projected distribution is taste-dependent
  2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
  3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2 \mathrm{Var}(t)$
  4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \;\Leftrightarrow\; t_i \geq t_j$
  5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$

  1. Projected distribution is taste-dependent
  2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
  3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2 \, \mathrm{Var}(t)$
  4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \;\Leftrightarrow\; t_i \geq t_j$
  5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

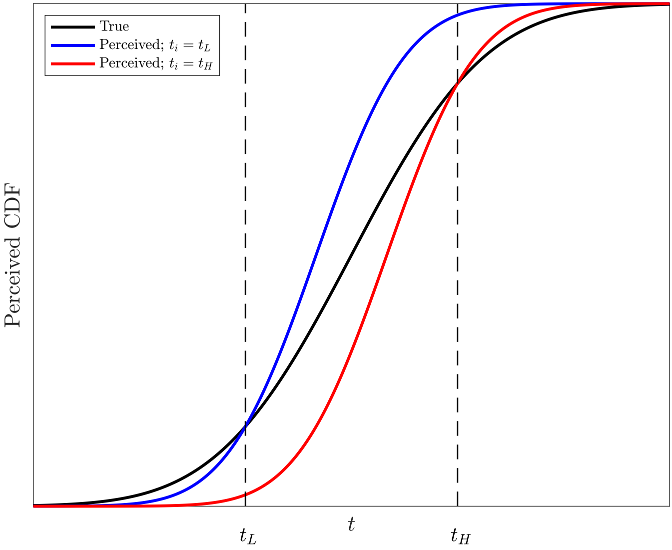
Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$
  1. Projected distribution is taste-dependent
  2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
  3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2 \, \mathrm{Var}(t)$
  4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \iff t_i \geq t_j$
  5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

Taste Projection: Parametric Model

- Let $\widehat{F}_i$ denote CDF of $\hat{t}(t_i)$
  1. Projected distribution is taste-dependent
  2. $\widehat{\mathbb{E}}_i[\hat{t}] = \alpha t_i + (1-\alpha)\mathbb{E}[t]$
  3. $\mathrm{Var}(\hat{t}) = (1-\alpha)^2 \, \mathrm{Var}(t)$
  4. $\widehat{F}_i \succsim_{FOSD} \widehat{F}_j \;\Leftrightarrow\; t_i \geq t_j$
  5. $\widehat{F}_i$ is a rotation of F about t_i: $\begin{cases} \widehat{F}_i(z) > F(z) & \text{if } z > t_i \\ \widehat{F}_i(z) = F(z) & \text{if } z = t_i \\ \widehat{F}_i(z) < F(z) & \text{if } z < t_i \end{cases}$

Taste Projection: Parametric Model



Taste Projection: Solution Concept

- **Naive projection:** Each player i plays a strategy that is optimal in perceived game where he (wrongly) assumes common knowledge that private tastes distributed according to $\widehat{F}_i$
  - $\Rightarrow$ Under-appreciates that others have different perceptions
- Let $\Gamma(F)$ denote game as function of distribution of private tastes
  - Player i wrongly thinks the game is $\Gamma(\widehat{F}_i)$
  - Wrong perception of taste distribution but correct about all else

Definition

Strategy profile $\hat{\beta} \equiv \left(\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_N\right)$ is *Naive Bayesian Equilibrium* if for all i, $\exists$ Bayesian Nash Equilibrium (BNE) $\tilde{\beta}$ of $\Gamma(\widehat{F}_i)$ s.t. $\tilde{\beta}_i = \hat{\beta}_i$.

- Each $\hat{\beta}_i$ is BNE strategy in the perceived game $\Gamma(\widehat{F}_i)$
- In truth, nobody's strategy is a best response to true behavior

Taste Projection: Solution Concept

- **Naive projection:** Each player i plays a strategy that is optimal in perceived game where he (wrongly) assumes common knowledge that private tastes distributed according to $\widehat{F}_i$
  - $\Rightarrow$ Under-appreciates that others have different perceptions
- Let $\Gamma(F)$ denote game as function of distribution of private tastes
  - Player i wrongly thinks the game is $\Gamma(\widehat{F}_i)$
  - Wrong perception of taste distribution but correct about all else

Definition

Strategy profile $\hat{\beta} \equiv \left(\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_N\right)$ is *Naive Bayesian Equilibrium* if for all i, $\exists$ Bayesian Nash Equilibrium (BNE) $\tilde{\beta}$ of $\Gamma(\widehat{F}_i)$ s.t. $\tilde{\beta}_i = \hat{\beta}_i$.

- Each $\hat{\beta}_i$ is BNE strategy in the perceived game $\Gamma(\widehat{F}_i)$
- In truth, nobody's strategy is a best response to true behavior

Taste Projection: Solution Concept

- **Naive projection:** Each player i plays a strategy that is optimal in perceived game where he (wrongly) assumes common knowledge that private tastes distributed according to $\widehat{F}_i$

  $\Rightarrow$ Under-appreciates that others have different perceptions

- Let $\Gamma(F)$ denote game as function of distribution of private tastes
  - Player i wrongly thinks the game is $\Gamma(\widehat{F}_i)$
  - Wrong perception of taste distribution but correct about all else

Definition

Strategy profile $\hat{\beta} \equiv \left(\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_N\right)$ is *Naive Bayesian Equilibrium* if for all i, $\exists$ Bayesian Nash Equilibrium (BNE) $\tilde{\beta}$ of $\Gamma(\widehat{F}_i)$ s.t. $\tilde{\beta}_i = \hat{\beta}_i$.

- Each $\hat{\beta}_i$ is BNE strategy in the perceived game $\Gamma(\widehat{F}_i)$
- In truth, nobody's strategy is a best response to true behavior

Private Values

- Let $\gamma = 0$. Then

$$V_i = t_i$$

- **Second-price (and English):** it is still a (weakly) dominant strategy for bidders to bid their value
- Intuition: my beliefs about the distribution of my opponents' values do not affect my incentives
- Matters are different in first-price auctions...

Private Values

- Let $\gamma = 0$. Then
$$V_i = t_i$$
- **Second-price (and English):** it is still a (weakly) dominant strategy for bidders to bid their value
- Intuition: my beliefs about the distribution of my opponents' values do not affect my incentives
- Matters are different in first-price auctions...

Private Values

- Let $\gamma = 0$. Then

$$V_i = t_i$$

- **Second-price (and English):** it is still a (weakly) dominant strategy for bidders to bid their value
- Intuition: my beliefs about the distribution of my opponents' values do not affect my incentives
- Matters are different in first-price auctions...

Private Values: First-Price Auctions

- In a first-price auction, unbiased bidder i bids

$$\beta_I^*(t_i) = \mathbb{E}\left[t^1 | t^1 < t_i\right] = \frac{\int_{\underline{t}}^{t_i} x f^1(x)\, dx}{F^1(t_i)}$$

where t^1 is value of strongest opponent

- Under taste projection, bidder i bids

$$\widehat{\beta}_I(t_i) = \widehat{\mathbb{E}}_i\left[t^1 | t^1 < t_i\right] = \frac{\int_{\alpha t_i + (1-\alpha)\underline{t}}^{t_i} x \widehat{f}_i^1(x)\, dx}{\widehat{F}_i^1(t_i)}$$

Proposition

1. *For all $t > \underline{t}$, $\widehat{\beta}_I(t) > \beta_I^*(t)$ and $\widehat{\beta}_I(t)$ is strictly increasing in α*
2. *Expected revenue is strictly increasing in α*

Private Values: First-Price Auctions

- In a first-price auction, unbiased bidder i bids

$$\beta_I^*(t_i) = \mathbb{E}\left[t^1 | t^1 < t_i\right] = \frac{\int_{\underline{t}}^{t_i} x f^1(x)\,dx}{F^1(t_i)}$$

where t^1 is value of strongest opponent

- Under taste projection, bidder i bids

$$\widehat{\beta}_I(t_i) = \widehat{\mathbb{E}}_i\left[t^1 | t^1 < t_i\right] = \frac{\int_{\alpha t_i + (1-\alpha)\underline{t}}^{t_i} x \widehat{f}_i^1(x)\,dx}{\widehat{F}_i^1(t_i)}$$

Proposition

1. *For all $t > \underline{t}$, $\widehat{\beta}_I(t) > \beta_I^*(t)$ and $\widehat{\beta}_I(t)$ is strictly increasing in α*
2. *Expected revenue is strictly increasing in α*

Private Values: First-Price Auctions

- In a first-price auction, unbiased bidder i bids

$$\beta_I^*(t_i) = \mathbb{E}\left[t^1 | t^1 < t_i\right] = \frac{\int_{\underline{t}}^{t_i} x f^1(x)\, dx}{F^1(t_i)}$$

where t^1 is value of strongest opponent

- Under taste projection, bidder i bids

$$\widehat{\beta}_I(t_i) = \widehat{\mathbb{E}}_i\left[t^1 | t^1 < t_i\right] = \frac{\int_{\alpha t_i + (1-\alpha)\underline{t}}^{t_i} x \widehat{f}_i^1(x)\, dx}{\widehat{F}_i^1(t_i)}$$

Proposition

1. *For all* $t > \underline{t}$, $\widehat{\beta}_I(t) > \beta_I^*(t)$ *and* $\widehat{\beta}_I(t)$ *is strictly increasing in* α
2. *Expected revenue is strictly increasing in* α

Private Values: First-Price Auctions

- Projection $\Rightarrow$ perceive a less-dispersed distribution of values
  - $\Rightarrow$ overestimate competition
  - $\Rightarrow$ Shade too little and bid too aggressively
  - $\Rightarrow$ FPA generates greater expected revenue than SPA
- Model predicts that *all* types overbid
  - A low-value player thinks others tend to be low-value as well
  - But he still overestimates value of competitor just below himself
  - Different from the case of affiliated values

Private Values: First-Price Auctions

- Projection $\Rightarrow$ perceive a less-dispersed distribution of values
  - $\Rightarrow$ overestimate competition
  - $\Rightarrow$ Shade too little and bid too aggressively
  - $\Rightarrow$ FPA generates greater expected revenue than SPA
- Model predicts that *all* types overbid
  - A low-value player thinks others tend to be low-value as well
  - But he still overestimates value of competitor just below himself
  - Different from the case of affiliated values

Private Values: First-Price Auctions

- Projection $\Rightarrow$ perceive a less-dispersed distribution of values
  - $\Rightarrow$ overestimate competition
  - $\Rightarrow$ Shade too little and bid too aggressively
  - $\Rightarrow$ FPA generates greater expected revenue than SPA
- Model predicts that *all* types overbid
  - A low-value player thinks others tend to be low-value as well
  - But he still overestimates value of competitor just below himself
  - Different from the case of affiliated values

Private Values: First-Price Auctions

- Projection ⇒ perceive a less-dispersed distribution of values
  - ⇒ overestimate competition
  - ⇒ Shade too little and bid too aggressively
  - ⇒ FPA generates greater expected revenue than SPA
- Model predicts that *all* types overbid
  - A low-value player thinks others tend to be low-value as well
  - But he still overestimates value of competitor just below himself
  - Different from the case of affiliated values

Private + Common Values

- For simplicity, suppose $N = 2$. Hence,

$$V_1 = \underbrace{t_1 + \gamma\theta_1}_{:=s_1} + \gamma\theta_2$$

$$V_2 = \underbrace{t_2 + \gamma\theta_2}_{:=s_2} + \gamma\theta_1$$

- Bidder i only observes t_i and θ_i, but her value also depends on θ_j
- Signal-extraction problem: i infers the value of θ_j from j's strategy, which in turn depends on s_j
- Bidders project only t_i

Private + Common Values

- For simplicity, suppose $N = 2$. Hence,

$$V_1 = \underbrace{t_1 + \gamma\theta_1}_{:=s_1} + \gamma\theta_2$$

$$V_2 = \underbrace{t_2 + \gamma\theta_2}_{:=s_2} + \gamma\theta_1$$

- Bidder i only observes t_i and θ_i, but her value also depends on θ_j
- Signal-extraction problem: i infers the value of θ_j from j's strategy, which in turn depends on s_j
- Bidders project only t_i

Sketch of the analysis of the SPA

- In equilibrium, unbiased bidder i bids

$$\beta_{II}^{*}(s_i) = \mathbb{E}[V_i | s_j = s_i] = s_i + \gamma \mathbb{E}[\theta_j | s_j = s_i]$$

- With taste projection, bidder i bids

$$\widehat{\beta}_{II}(s_i | t_i) = \widehat{\mathbb{E}}_i[V_i | s_j = s_i] = s_i + \gamma \widehat{\mathbb{E}}_i[\theta_j | s_j = s_i]$$

- Projection distorts i's inference about θ_j: misinference effect

Lemma

For all interior k, $\widehat{\mathbb{E}}_i[\theta_j | s_j = k]$ is strictly decreasing in t_i.

Sketch of the analysis of the SPA

- In equilibrium, unbiased bidder i bids

$$\beta_{II}^{*}(s_i) = \mathbb{E}[V_i | s_j = s_i] = s_i + \gamma \mathbb{E}[\theta_j | s_j = s_i]$$

- With taste projection, bidder i bids

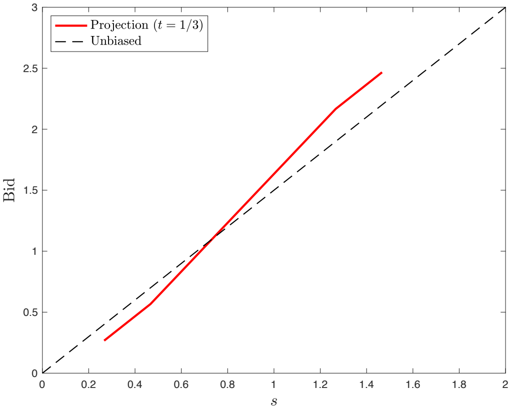
$$\widehat{\beta}_{II}(s_i | t_i) = \widehat{\mathbb{E}}_i [V_i | s_j = s_i] = s_i + \gamma \widehat{\mathbb{E}}_i [\theta_j | s_j = s_i]$$

- Projection distorts i's inference about θ_j: misinference effect

Lemma

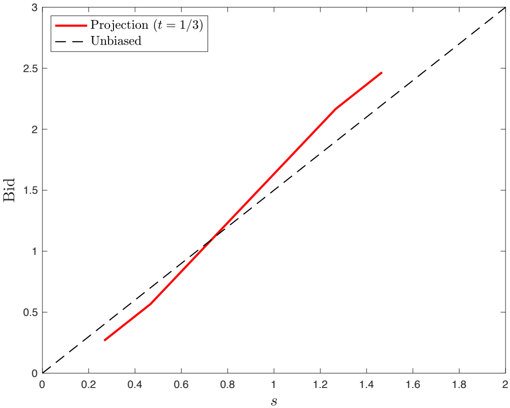
For all interior k, $\widehat{\mathbb{E}}_i [\theta_j | s_j = k]$ is strictly decreasing in t_i.

SPA Bidding Strategy



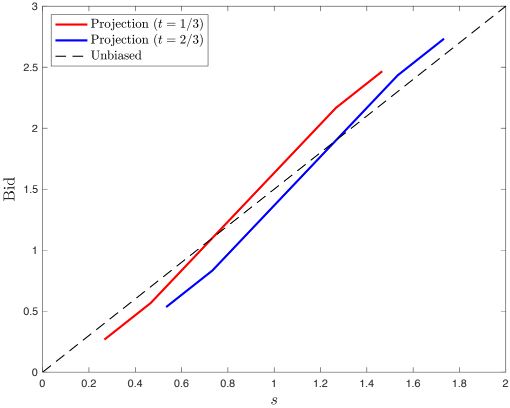
- Holding t_i fixed, $\widehat{\beta}(s|t_i)$ too sensitive to common-value signal:
  - Strategy conditions on $s_j = t_i + \theta_i$
  - high $s_i \Rightarrow \widehat{\mathbb{E}}_i [\theta_j | s_j = s_i] > \mathbb{E}_i [\theta_j | s_j = s_i]$
  - low $s_i \Rightarrow \widehat{\mathbb{E}}_i [\theta_j | s_j = s_i] < \mathbb{E}_i [\theta_j | s_j = s_i]$

SPA Bidding Strategy



- Holding t_i fixed, $\widehat{\beta}(s|t_i)$ too sensitive to common-value signal:
  - Strategy conditions on $s_j = t_i + \theta_i$
  - high $s_i \Rightarrow \widehat{\mathbb{E}}_i\left[\theta_j|s_j = s_i\right] > \mathbb{E}_i\left[\theta_j|s_j = s_i\right]$
  - low $s_i \Rightarrow \widehat{\mathbb{E}}_i\left[\theta_j|s_j = s_i\right] < \mathbb{E}_i\left[\theta_j|s_j = s_i\right]$

SPA Bidding Strategy



- Holding s_i constant, $\widehat{\beta}_{II}\left(s_i \middle| t_i\right)$ is strictly decreasing in t_i:
  - High $t_i \Rightarrow$ under-estimate θ_j
  - Low $t_i \Rightarrow$ over-estimate θ_j

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$
  - Efficient outcome iff $t_i > t_j$

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$
- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j) \Leftrightarrow t_i < t_j$

  $\Rightarrow$ There exist realizations where $s_i < s_j$ but $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ If projection changes winner, always less efficient outcome

  - Projection makes bidders with high private tastes *less* optimistic about the common value — but these are the efficient winners!

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$
  - Efficient outcome iff $t_i > t_j$

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$

- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j) \Leftrightarrow t_i < t_j$

  $\Rightarrow$ There exist realizations where $s_i < s_j$ but $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ If projection changes winner, always less efficient outcome

  - Projection makes bidders with high private tastes *less* optimistic about the common value — but these are the efficient winners!

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$
  - Efficient outcome iff $t_i > t_j$

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$

- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j) \Leftrightarrow t_i < t_j$

  $\Rightarrow$ There exist realizations where $s_i < s_j$ but $\widehat{\beta}_{II}(s_i|t_i) > \widehat{\beta}_{II}(s_j|t_j)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ If projection changes winner, always less efficient outcome

  - Projection makes bidders with high private tastes *less* optimistic about the common value — but these are the efficient winners!

Sketch of the analysis of the FPA

- In equilibrium, unbiased bidder i bids

$$\beta_I^*(s_i) = \mathbb{E}[V_i|s_j < s_i] - \mathbb{E}[s_i - s_j|s_j < s_i] = \gamma\mathbb{E}[\theta_j|s_j < s_i] + \mathbb{E}[s_j|s_j < s_i]$$

- With taste projection, bidder i bids

$$\widehat{\beta}_I(s_i|t_i) = \widehat{\mathbb{E}}_i[V_i|s_j < s_i] - \widehat{\mathbb{E}}_i[s_i - s_j|s_j < s_i] = \gamma\widehat{\mathbb{E}}_i[\theta_j|s_j < s_i] + \widehat{\mathbb{E}}_i[s_j|s_j < s_i]$$

Projection distorts i's inference about θ_j and s_j:

Lemma (Misinference Effect)

For all interior k, $\widehat{\mathbb{E}}_i[\theta_j|s_j < k]$ is strictly decreasing in t_i.

Lemma (Competition Effect)

For all interior k, $\widehat{\mathbb{E}}_i[s_j|s_j < k]$ is strictly increasing in t_i.

Sketch of the analysis of the FPA

- In equilibrium, unbiased bidder i bids

$$\beta_I^*(s_i) = \mathbb{E}[V_i|s_j < s_i] - \mathbb{E}[s_i - s_j|s_j < s_i] = \gamma\mathbb{E}[\theta_j|s_j < s_i] + \mathbb{E}[s_j|s_j < s_i]$$

- With taste projection, bidder i bids

$$\widehat{\beta}_I(s_i|t_i) = \widehat{\mathbb{E}}_i[V_i|s_j < s_i] - \widehat{\mathbb{E}}_i[s_i - s_j|s_j < s_i] = \gamma\widehat{\mathbb{E}}_i[\theta_j|s_j < s_i] + \widehat{\mathbb{E}}_i[s_j|s_j < s_i]$$

Projection distorts i's inference about θ_j and s_j:

Lemma (Misinference Effect)

For all interior k, $\widehat{\mathbb{E}}_i[\theta_j|s_j < k]$ is strictly decreasing in t_i.

Lemma (Competition Effect)

For all interior k, $\widehat{\mathbb{E}}_i[s_j|s_j < k]$ is strictly increasing in t_i.

Sketch of the analysis of the FPA

- In equilibrium, unbiased bidder i bids

$$\beta_I^*(s_i) = \mathbb{E}[V_i|s_j < s_i] - \mathbb{E}[s_i - s_j|s_j < s_i] = \gamma\mathbb{E}[\theta_j|s_j < s_i] + \mathbb{E}[s_j|s_j < s_i]$$

- With taste projection, bidder i bids

$$\widehat{\beta}_I(s_i|t_i) = \widehat{\mathbb{E}}_i[V_i|s_j < s_i] - \widehat{\mathbb{E}}_i[s_i - s_j|s_j < s_i] = \gamma\widehat{\mathbb{E}}_i[\theta_j|s_j < s_i] + \widehat{\mathbb{E}}_i[s_j|s_j < s_i]$$

Projection distorts i's inference about θ_j and s_j:

Lemma (Misinference Effect)

For all interior k, $\widehat{\mathbb{E}}_i[\theta_j|s_j < k]$ is strictly decreasing in t_i.

Lemma (Competition Effect)

For all interior k, $\widehat{\mathbb{E}}_i[s_j|s_j < k]$ is strictly increasing in t_i.

Sketch of the analysis of the FPA

- In equilibrium, unbiased bidder i bids

$$\beta_I^*(s_i) = \mathbb{E}\left[V_i|s_j < s_i\right] - \mathbb{E}\left[s_i - s_j|s_j < s_i\right] = \gamma\mathbb{E}\left[\theta_j|s_j < s_i\right] + \mathbb{E}\left[s_j|s_j < s_i\right]$$

- With taste projection, bidder i bids

$$\widehat{\beta}_I(s_i|t_i) = \widehat{\mathbb{E}}_i\left[V_i|s_j < s_i\right] - \widehat{\mathbb{E}}_i\left[s_i - s_j|s_j < s_i\right] = \gamma\widehat{\mathbb{E}}_i\left[\theta_j|s_j < s_i\right] + \widehat{\mathbb{E}}_i\left[s_j|s_j < s_i\right]$$

Projection distorts i's inference about θ_j and s_j:

Lemma (Misinference Effect)

For all interior k, $\widehat{\mathbb{E}}_i\left[\theta_j|s_j < k\right]$ is strictly decreasing in t_i.

Lemma (Competition Effect)

For all interior k, $\widehat{\mathbb{E}}_i\left[s_j|s_j < k\right]$ is strictly increasing in t_i.

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$.
  - Efficient outcome iff $t_i > t_j$.
  - $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$
- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_I(s_i|t_i) \lesseqgtr \widehat{\beta}_I(s_j|t_j)$ depends on relative strength of misinference vs. competition effects
  - Again, exist realizations where $s_i < s_j$ but $\widehat{\beta}_I(s_i|t_i) > \widehat{\beta}_I(s_j|t_j)$
  - $\Rightarrow$ Projection can lead to different winner than rational auction
  - $\Rightarrow$ Change in winner can improve efficiency (due to competition effect)
    - Sufficient condition: $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[s_j|s_j < k] + \gamma\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j|s_j < k] < 1$
  - Projection makes bidders with high private tastes *more* concerned about competition — and these are the efficient winners!

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$.
  - Efficient outcome iff $t_i > t_j$.

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$

- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_I\left(s_i|t_i\right) \lesseqgtr \widehat{\beta}_I\left(s_j|t_j\right)$ depends on relative strength of misinference vs. competition effects
  - Again, exist realizations where $s_i < s_j$ but $\widehat{\beta}_I\left(s_i|t_i\right) > \widehat{\beta}_I\left(s_j|t_j\right)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ Change in winner can improve efficiency (due to competition effect)

    - Sufficient condition: $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[s_j|s_j < k] + \gamma\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j|s_j < k] < 1$

  - Projection makes bidders with high private tastes *more* concerned about competition — and these are the efficient winners!

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$.
  - Efficient outcome iff $t_i > t_j$.

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$
- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_I(s_i|t_i) \lessgtr \widehat{\beta}_I(s_j|t_j)$ depends on relative strength of misinference vs. competition effects
  - Again, exist realizations where $s_i < s_j$ but $\widehat{\beta}_I(s_i|t_i) > \widehat{\beta}_I(s_j|t_j)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ Change in winner can improve efficiency (due to competition effect)
    - Sufficient condition: $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[s_j|s_j < k] + \gamma\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j|s_j < k] < 1$
  - Projection makes bidders with high private tastes *more* concerned about competition — and these are the efficient winners!

(In)efficiency

- **Unbiased benchmark:**
  - i wins iff $s_i > s_j$.
  - Efficient outcome iff $t_i > t_j$.

  $\Rightarrow$ Inefficient outcome if $s_i > s_j$ and $t_i < t_j$
- **Projection:**
  - If $s_i = s_j$, then $\widehat{\beta}_I(s_i|t_i) \lesseqgtr \widehat{\beta}_I(s_j|t_j)$ depends on relative strength of misinference vs. competition effects
  - Again, exist realizations where $s_i < s_j$ but $\widehat{\beta}_I(s_i|t_i) > \widehat{\beta}_I(s_j|t_j)$

  $\Rightarrow$ Projection can lead to different winner than rational auction

  $\Rightarrow$ Change in winner can improve efficiency (due to competition effect)
    - Sufficient condition: $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[s_j|s_j < k] + \gamma\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j|s_j < k] < 1$
  - Projection makes bidders with high private tastes *more* concerned about competition — and these are the efficient winners!

Efficiency Comparison

- In SPA, projection lowers efficiency compared to the benchmark
- In FPA, projection may increase efficiency compared to the benchmark

Proposition

Suppose $\alpha > 0$. *If* $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j | s_j < k] \leq \frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j | s_j = k]$, *then the FPA is more efficient than the SPA.*

- Meaning of sufficient condition: inference about opponent's information more sensitive to conditioning on opponents' precise interim value rather than conditioning on falling below that value
  - $\Rightarrow$ Pessimistic misinference is stronger in SPA than FPA

Efficiency Comparison

- In SPA, projection lowers efficiency compared to the benchmark
- In FPA, projection may increase efficiency compared to the benchmark

Proposition

Suppose $\alpha > 0$. If $\frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j | s_j < k] \leq \frac{\partial}{\partial k}\widehat{\mathbb{E}}_i[\theta_j | s_j = k]$, then the FPA is more efficient than the SPA.

- Meaning of sufficient condition: inference about opponent's information more sensitive to conditioning on opponents' precise interim value rather than conditioning on falling below that value
  - $\Rightarrow$ Pessimistic misinference is stronger in SPA than FPA

Extensions

1. Optimal Reserve Price w/ IPV.
2. Asymmetric Auctions w/ Private Values la Maskin and Riley (2000)
3. English Auction w/ Private and Common Values and $N > 2$

Conclusions

- Model capturing the implications of taste projection in auctions
- **With IPV:**
  - Projection provides a novel explanation for overbidding in FPA
  - FPA fetches higher revenue than SPA
- **With a common-value element:**
  - Misinference effect vs. competition effect
  - FPA more efficient than SPA

Conclusions

- Model capturing the implications of taste projection in auctions
- **With IPV:**
  - Projection provides a novel explanation for overbidding in FPA
  - FPA fetches higher revenue than SPA
- **With a common-value element:**
  - Misinference effect vs. competition effect
  - FPA more efficient than SPA