Parallel Innovation Contests

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Motivation and Questions

Innovation Contests in Practice

- Organizers posts contests, agents develop and submit innovative solutions, organizers reward the best solutions.
- Many organizations run multiple contests
  - Elanco; Bill and Melinda Gates Foundation; Challenge.gov

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- Contests (crowdsourcing) platforms
  - Topcoder, InnoCentive, Ennomotive
- Users of these platforms
  - HP, Ford, NASA, P&G
- Most organizations/platforms determine/instruct awards and terms and conditions in all of their contests
- Multiple contests run in the platform and platform members (agents) work on these contests

Entering Multiple Contests: InnoCentive vs TopCoder

- InnoCentive
  - Solicits innovative solutions
  - Subject categories such as biology and business
  - (Interview) Agents freely work on multiple contests
  - (Interview) 57% of agents work on more than one contest within a day
- TopCoder
  - Solicits software solutions
  - (Interview) Working on multiple contests discouraged in lower-novelty development challenges (e.g., fixing bugs)
  - (Interview) Agents work on multiple algorithm contests that require novel solutions (e.g., 2D image - 3D object matching)

Research Questions

- Q1) Should agents be allowed to freely participate in multiple contests or should they be discouraged?
- Q2) How do multiple contests affect organizers? Should a company organize multiple contests? How many contests should organizers run?

Related Literature

- Single Organizer & Heterogeneous Agents + No Output Uncertainty
  - Moldovanu & Sela (AER, 2001; JET, 2006) and many others
- Multiple Organizers & Heterogeneous Agents + No Output Uncertainty
  - Azmat & Moller (RAND, 2009); Buyukboyaci (EI, 2016); Hafalir et al. (2018)
- Single Organizer & Homogeneous Agents + Output Uncertainty
  - Lazear and Rosen (1981);Terwiesch & Xu (MS, 2008); Ales, Cho, & Korpeoglu (2016) and others

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- Multiple Organizers & Homogeneous Agents + Output Uncertainty
  - This paper

Model

Model: Sequence of Events

1. Organizers announce winner award for own contests anticipating the awards of other contests
2. Agents decide (i) which contest(s) to participate in and (ii) effort at each contest
3. Agents create *stochastic* outputs depending on efforts
4. Organizers collect outputs and award best output

Model: Decisions of Organizers and Agents

- M organizers and contests ($m \in \{1, 2, ..., M\}$)
- Winner award A_m in contest m
- N agents ($i \in \{1, 2, ..., N\}$)
- Effort e_{im} in contest $m \rightarrow$ equilibrium effort e_m^*
- Output $y_{im} = r(e_{im}) + \widetilde{\xi}_{im}$
  - r is increasing and concave (r' is HD$-k$)
  - ξ is the error term with zero mean and logconcave density $H' = h$

Model: Objectives of Organizers and Agents

- Decentralized organizers
  - Organizer m chooses A_m to maximize $\Pi_m = \max_i \; y_{im} - A_m$
- Centralized organizers
  - Planner chooses $(A_1, ..., A_M)$ to maximize $\overline{\Pi} = \frac{1}{M} \sum_{m=1}^{M} \Pi_m$

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- Agents
  - Utility: $\sum_{m=1}^{M} A_m P\{y_{im} = \max_j y_{jm}\} - \psi(\sum_{m=1}^{M} e_{im})$
  - Agents' cost function is of the form

$$\psi(e_{i1}, e_{i2}, ..., e_{iM}) = \eta \left(\sum_{m=1}^{M} \phi(e_{im}) \right),$$

where η is increasing and HDb (< 1), ϕ is increasing and HDp (> 1); so that $\frac{\partial \psi}{\partial e_{im}} > 0$; $\frac{\partial^2 \psi}{\partial e_{im}^2} \geq 0$; and $\frac{\partial^2 \psi}{\partial e_{il} \partial e_{im}} < 0$ for all $l \neq m$.

Equilibrium

Equilibrium Among Agents I

- We focus on symmetric pure strategy Nash equilibrium

$$P_m(e_{im}, e_m^*) = \int_{s \in \Xi} H(s + r(e_{im}) - r(e_m^*))^{N-1} h(s) ds.$$

- Agent's problem:

$$\max_{(e_{i1}, e_{i2}, \ldots, e_{iM})} \sum_{m=1}^{M} A_m P_m(e_{im}, e_m^*) - \psi\left(e_{i1}, e_{i2}, ..., e_{iM}\right) \text{ s.t. } \sum_{m=1}^{M} e_{im} \le \overline{E}.$$

Equilibrium Among Agents II

- For $I_N \equiv \int_{s\in\Xi}(N-1)H(s)^{N-2}h(s)^2 ds$, denote the solution to

$$A_m r'(\widehat{e}_m) I_N = \eta'\left(\sum_{l=1}^{M}\phi(\widehat{e}_l)\right)\phi'(\widehat{e}_m) \text{ for all } m \in \{1,2,\ldots,M\}$$

by $\widehat{e}_m$; and the solution to

$$A_m r'(e_m^*) I_N - \eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_m^*) = A_1 r'(e_1^*) I_N - \eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_1^*)$$

$$\text{and } \sum_{m=1}^{M} e_m^* = \overline{E}.$$

by e_m^*

Lemma

When $\sum_{m=1}^{M}\widehat{e}_m < \overline{E}$, the unique symmetric-equilibrium effort at contest m is $e_m^* = \widehat{e}_m$, and when $\sum_{m=1}^{M}\widehat{e}_m \geq \overline{E}$, the unique symmetric-equilibrium effort is e_m^*.

Coordinator's problem

- Coordinator's objective is to maximize

$$\overline{\Pi} = \frac{\sum_{m=1}^{M} r(e_m^*)}{M} + \frac{E\left[\sum_{m=1}^{M} \widetilde{\xi}_{(1)m}^{N}\right]}{M} - \frac{\sum_{m=1}^{M} A_m}{M}.$$

- Define $g(x) = ((\eta \circ \phi)'/r')^{-1}(x)$;
  $\Phi(A) = r'(e^*)\, g'\left(A I_N M^{1-b}\right) I_N M^{1-b} - 1$ and
  $\overline{A} = M^{b-1} g^{-1}\left(\overline{E}/M\right)/I_N$.

Lemma

If $\Phi(\overline{A}) \geq 0$, then $\overline{\Pi}$ is maximized at $A_m^* = A^* = \overline{A}$ and $e_m^* = e^* = \overline{E}/M$. If $\Phi(\overline{A}) < 0$, then there exists a unique $\widehat{A}$ such that $\Phi(\widehat{A}) = 0$, and $\overline{\Pi}$ is maximized at $A_m^* = A^* = \widehat{A}$ and $e_m^* = e^* = g(A^* I_N M^{1-b})$.

Scale Transformation

Definition

For any distribution $H(s)$ and density $h(s)$, $\widehat{H}(s) = H(s/\alpha)$ and $\widehat{h}(s) = h(s/\alpha)/\alpha$ are called α-scale transformation. After transformation, output shock becomes $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$.

Variance of $\widehat{\xi}_{im} = \alpha^2 \times$ Variance of $\widetilde{\xi}_{im}$

Results

(Q1) Exclusive vs Non-Exclusive Contests

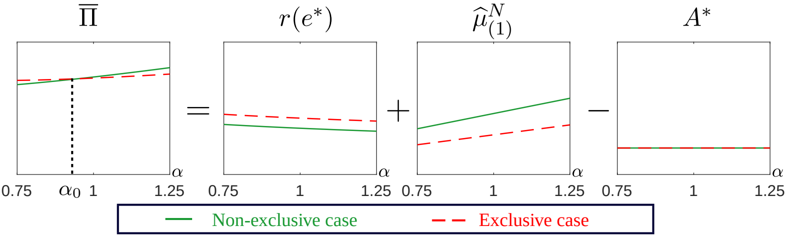
- Q1: Should agents be allowed to freely participate in multiple contests or should they be discouraged?
- Non-exclusive: agents can freely participate in multiple contests (our previous result assumed so).
- Exclusive: each agent can participate in only one contest
- $\overline{\Pi}^X$: The average profit of organizers under exclusive case when a planner determines the optimal allocation of agents and awards at all contests.

Theorem

Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. There exists α_0 such that the average profit in the non-exclusive case $\overline{\Pi}$ is greater than that in the exclusive case $\overline{\Pi}^X$ for any $\alpha > \alpha_0$.

(Q1) Exclusivity vs Non-Exclusivity: Intuition and Insight

- Intuition: Higher effort in exclusive but higher diversity in non-exclusive



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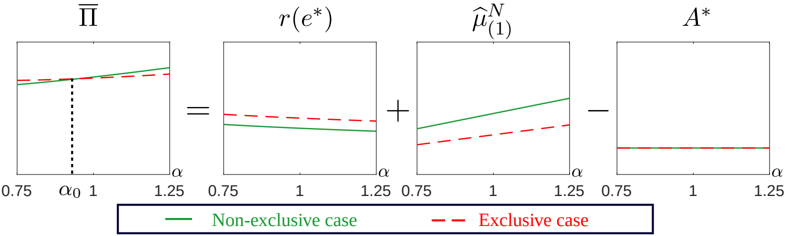

- Insight:
  - Discourage participation in multiple contests that seek lower novelty solutions (TopCoder)
  - Encourage participation in multiple contests that seek major innovation (InnoCentive)

(Q2) Optimal Number of Contests

How does $\overline{\Pi}$ change with the number of contests M? How does optimal number of contests change with output uncertainty?

Theorem

The average profit $\overline{\Pi}$ is unimodal in the number of contests M, i.e., there exists $M^* \in [1, \infty)$ such that $\frac{\partial \overline{\Pi}}{\partial M} > 0$ for all $M < M^*$ and $\frac{\partial \overline{\Pi}}{\partial M} < 0$ for all $M > M^*$.

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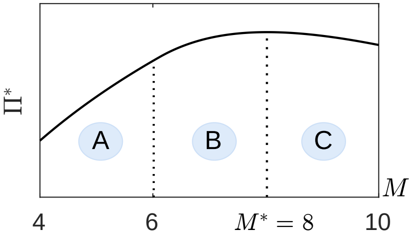
Lemma

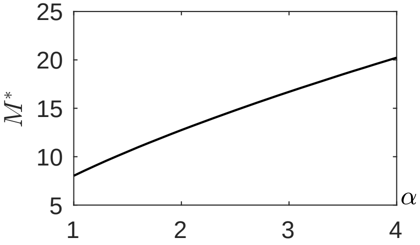
Suppose that output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha \widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, M^* is increasing in α.

(Q2) Optimal Number of Contests, Intuition I



- Two effects: Scope effect due to economies of scope and scarcity effect to due less effort when agent's capacity binds
- M values where there is no scarcity effect (region A), there is a scarcity effect but it is dominated by the scope effect (region B), and the scope effect is dominated by the scarcity effect (region C)

(Q2) Optimal Number of Contests, Intuition II



- As α increases, agent reduce her effort which leads to a smaller scarcity effect and a smaller scope effect.
- The scarcity effect decreases with the agent's output uncertainty more than the scope effect, and hence the scope effect outweighs the scarcity effect up to a larger number of contests M^*.

(Q2) Optimal Number of Contests, Insights

- There is a bound on the number of beneficial contests due to scarcity of resources
- Optimal M is larger when α is large (e.g. when seeking novel solutions.)

Further Results

Fixed Cost of Participation

- Consider the case where each agent incurs a fixed cost c_f for each contest she participates (no capacity constraint). Agent's utility from participating in M contests

$$U[M] = \frac{AM}{N} - M^b \eta(\phi(e^*)) - Mc_f,$$

Proposition

(a) Suppose $k \geq 1$. The agent's participation condition holds for any M.
(b) Suppose $k < 1$, and that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, there exists a unique $\overline{M}$ such that the agent's participation condition is violated when $M > \overline{M}$. Also, $\overline{M}$ is increasing in α.

Agents can participate in a limited number of contests.

- We consider N agents where each agent enters a single contest and compare profits when N agents enter a single contest with that of N_1 agents enter one contest and $N_2(= N - N_1)$ agents enter the other contest.

Proposition

Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Two contests with N_1 and N_2 agents yield a larger average profit $\overline{\Pi}^L$ than a single contest with $N_1 + N_2$ agents if and only if $\alpha < \alpha_L \equiv \frac{\theta}{bp} \frac{\log\left(I_{N_1} I_{N_2}\right) - 2\log\left(I_{N_1+N_2}\right)}{2\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_1} - \mu_{(1)}^{N_2}}$.

Managerial Insights, Extensions and Conclusion

Managerial Insights

Table 1 The summary of key results and managerial insights.

	Small uncertainty (e.g., when seeking low-novelty solutions)	Large uncertainty (e.g., when seeking innovative solutions)
Exclusive vs non-exclusive contests	Exclusive contests are optimal (Theorem 1 and Corollary 1).	Non-exclusive contests are optimal (Theorem 1 and Corollary 1).
Multiple contests or not	Running more contests than what each agent can participate in improves each organizer's profit (Proposition 2).	Each organizer's profit increases with the number of contests up to an optimal number of contests (Theorem 2).
Managerial insights	Run multiple contests in parallel (up to a certain number) but discourage agents from participating in multiple contests.	Run multiple contests in parallel (up to a certain number) and encourage agents to participate in multiple contests.

Extensions

- We also consider
  - Alternative Objective for Coordinator (the sum of profits rather than average profit)
  - Decentralized Contests (where contest holders compete with each other)
  - Alternative Model for Economies of Scope (where efforts in other contests improve quality of a contest)

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- In these extensions, our main results continue to hold.

Conclusion

- When output uncertainty is large, the non-exclusive case generates larger profits for organizers than the exclusive case.
- An organizer's profit can increase up to an optimal number of contests, and the drivers for the optimal number of contests depend on the agent's output uncertainty
- Technical contribution: we are able to solve a model with parallel contests with (i) a cost function that exhibits diseconomies of scale at each contest and economies of scope across contests, and (ii) bounded efforts or cost of participation.

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- Technical contribution: we are able to solve a model with parallel contests with (i) a cost function that exhibits diseconomies of scale at each contest and economies of scope across contests, and (ii) bounded efforts or cost of participation.
- Our paper brings theory closer to the practice.

Future Directions

- Considering heterogeneous organizers
- Considering asymmetric equilibria
- How to dynamically schedule multiple contests, potentially considering the duration of each contest to maximize the profits

Thank you!