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Parallel Innovation Contests

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We study multiple parallel contests where contest organizers elicit solutions to innovation-related problems from a set of agents. Each agent may participate in multiple contests and exert effort to improve her solution for each contest she enters, but the quality of her solution also depends on an output uncertainty. We first analyze whether an organizer's profit can be improved by discouraging agents from participating in multiple contests. We show, interestingly, that organizers benefit from agents' participation in multiple contests when the agent's output uncertainty is sufficiently large. A managerial insight from this result is that when organizers elicit innovative solutions rather than low-novelty solutions, agents' participation in multiple contests may be beneficial to organizers. We further show that an organizer's profit is unimodal in the number of contests, and the optimal number of contests increases with the agent's output uncertainty. This finding may explain why many organizations run multiple contests in practice, and it prescribes a larger number of contests when organizations seek innovative solutions rather than low-novelty solutions.

Key words: Competition, Crowdsourcing, Platform, Tournament.

1. Introduction

With the advancements in information technology and the Internet, organizations have started to look beyond their boundaries in their search for innovation (Chesbrough 2003). For example, 85% of top global brands have used crowdsourcing in the last ten years (Chen et al. 2018). A popular method of crowdsourcing is the *innovation contest.* In an innovation contest, an organizer elicits innovative solutions to a challenging problem from a group of agents, and gives an award to the agent who submits the best solution. Because an innovation contest allows an organizer to tap into a large number of solutions at a low cost, the contest industry has evolved into a lucrative business where billions of dollars are awarded annually (McKinsey & Company 2009).

With the increased popularity of contests, crowdsourcing platforms such as InnoCentive and Topcoder now organize numerous contests, and generate $1 billion in revenue with an annual growth rate of 37.1% (Chen et al. 2018). For example, InnoCentive organizes around 200 contests annually for its customers in subject categories such as business, chemistry, and life sciences. These contests are often run in parallel, and InnoCentive members (agents) often participate in multiple contests and may win cash awards ranging from $5,000 to $1 million.[1] Similarly, Topcoder organizes around 6,000 software contests annually, and Topcoder members compete for awards around $10,000. Our interviews with practitioners at InnoCentive and Topcoder have revealed that a platform either determines contest rules (such as awards given to winners) on behalf of its customers or instructs its customers in setting these rules.[2] Also, a contest platform may encourage or discourage agents' participation in multiple contests by setting its terms and conditions accordingly.[3]

Besides contest platforms, many organizations run multiple contests in parallel. For instance, Elanco, an animal healthcare company, has organized five contests in 2016 that elicit innovative solutions to animal healthcare problems (Elanco 2017). Similarly, Bill and Melinda Gates Foundation (hereafter, Gates Foundation) has organized fourteen contests in 2016 within the Grand Challenges Explorations initiative, where agents develop innovative solutions to challenging healthcare problems. Most of these contests are run in parallel, providing agents with several problems to work on. Yet, some of these organizations discourage agents from participating in multiple contests. For instance, Gates Foundation allows submission to a single contest (GrandChallenges 2017).

Practitioners who run multiple contests need to make two important decisions. The first decision is whether to discourage agents from participating in multiple contests. If each agent enters only one contest, agents do not split their efforts among contests, so they may exert more effort at each contest. Indeed, the (economics) literature on multiple contests assumes that each agent enters only one contest (e.g., Azmat and Möller 2009). Yet, in practice, platforms such as InnoCentive allow agents to freely enter multiple contests. The second decision is how many contests to run in parallel. Running more contests may induce agents to split their efforts, so it may reduce the quality of solutions for each contest. In this paper, we generate insights into these decisions by answering the following research questions. (Q1) When should agents be discouraged from participating in multiple contests? (Q2) How does the number of contests affect an organizer's profit?

To answer these questions, we build a model of innovation contests where multiple contest organizers elicit solutions from a set of agents. After all awards are announced, each agent exerts effort to improve her solution for each contest she enters, where the quality of her solution also depends on an output uncertainty. Our model introduces new features that not only enable the analysis of the above research questions, but also contribute to the innovation contest theory. First, while the prior literature restricts attention to a single contest, we analyze multiple parallel contests. This analysis requires us to characterize a multidimensional optimization problem for each agent who chooses her effort at each contest by considering her total cost of effort. Second, while the prior literature assumes that an agent can exert unbounded effort, consistent with practice, we consider an agent's capacity constraint. Third, building on economics and operations literature, we factor in two effects that determine the shape of an agent's cost function: (i) each contest exhibits diseconomies of scale as it may be increasingly difficult for an agent to improve the quality of her solution for a certain contest (e.g., Mihm and Schlapp 2017), and (ii) there is a potential economies of scope across contests as exerting effort at one contest may reduce the cost of effort at another contest (e.g., Willig 1979, Panzar and Willig 1981).[4] While these novelties increase the complexity of our analysis and require special technical attention, they allow us to capture important aspects of innovation contests in practice.

We answer our first research question by comparing an "exclusive" case where each agent can participate in only one contest with a "non-exclusive" case where each agent can participate in multiple contests. We show that when agents face sufficiently large output uncertainty, an organizer's profit in the non-exclusive case is larger than that in the exclusive case. The intuition is as follows. While an exclusive contest incentivizes agents to exert more effort, a non-exclusive contest attracts a larger number of agents, and hence benefits from a more diverse set of solutions. The diversity effect outweighs the incentive effect when agents face sufficiently large output uncertainty. This result advises practitioners to run non-exclusive contests when they seek innovative solutions, and to run exclusive contests when they seek low-novelty solutions. For example, InnoCentive may maximize the outcome of theoretical challenges that seek innovative solutions (e.g., finding solutions to increase the literacy of deaf children in developing countries) by encouraging agents to participate in multiple contests. In contrast, Topcoder may maximize the outcome of development challenges that seek low-novelty solutions (e.g., finding bugs in a software) by discouraging agents from participating in multiple contests (e.g., restricting submissions similarly to Gates Foundation).

We next analyze how the number of contests affects an organizer's profit, and show that an organizer's profit can increase up to an optimal number of contests. Interestingly, the intuition of this result depends on the agent's output uncertainty. When the agent's output uncertainty is large, as discussed above, running non-exclusive contests maximizes each organizer's profit, and there is an optimal number of non-exclusive contests. This is because more non-exclusive contests may benefit organizers due to the economies-of-scope effect, but may also harm organizers because agents may split their efforts among more contests or they may even refrain from participating in some of these contests. We further show, interestingly, that the optimal number of contests increases with the agent's output uncertainty. This finding (along with its intuition) suggests that practitioners who seek innovative solutions may benefit from organizing multiple contests that exhibit economies of scope. When the agent's output uncertainty is small, running exclusive contests maximizes each organizer's profit. In the exclusive case, because each agent enters only one contest, the economies-of-scope effect disappears but a different trade-off arises. As the number of contests increases, the number of agents at each contest decreases, thereby incentivizing each agent to exert more effort, yet reducing the diversity of solutions. Thus, when the agent's output uncertainty is small (e.g., when organizers seek low-novelty solutions), the incentive effect outweighs the diversity effect, so running multiple contests improves each organizer's profit.

Related Literature. Our paper belongs to the literature on innovation contests and contributes to the scant literature on multiple contests.

Research on innovation contests within the operations literature is pioneered by Terwiesch and Xu (2008), who show that a free-entry open innovation contest is optimal. By generalizing Terwiesch and Xu (2008), Boudreau et al. (2011) show empirically and Ales et al. (2017b) show analytically that free-entry open innovation is optimal only when the agent's output uncertainty is sufficiently large. Similarly, building on the modeling framework of Terwiesch and Xu (2008), Nittala and Krishnan (2016) study the design of innovation contests within firms, Ales et al. (2017c) study the optimal set of awards in a contest, Mihm and Schlapp (2017) analyze whether and how to give feedback to agents, and Hu and Wang (2017) examine whether to run a single-stage or a sequential contest in the presence of multiple attributes.[5] Building on the modeling framework of these studies, we contribute to this literature in two ways. First, while these studies restrict attention to a single contest, we consider multiple contests. This multiple-contest environment helps us bridge the gap between theory and practice and contribute to the innovation contest theory by capturing novel features such as an agent's capacity constraint and economies of scope across contests. Second, we analyze novel research questions of when an organizer should discourage agents from participating in multiple contests and how the number of contests affects an organizer's profit.

Our paper contributes to the scant literature on multiple contests.[6] DiPalantino and Vojnović (2009) study multiple all-pay contests with exogenously given awards, and characterize equilibria for agents, but do not analyze the optimal decisions for organizers. Azmat and Möller (2009) consider two identical Tullock contests, and analyze the optimal award structure for organizers who compete for the participation of a set of identical agents. Büyükboyacı (2016) considers two agents where each agent exerts large or small effort, and compares running two parallel contests (potentially one agent in each contest) with running a single contest. Hafalır et al. (2016) compare running two all-pay contests with running a single all-pay contest, and focus on the equilibrium among agents without analyzing the optimal decisions for organizers.

It is noteworthy that the scant literature on multiple contests has provided only preliminary answers to some aspects of multiple contests. First, the above papers restrict attention to exclusive contests—an assumption often violated in practice—and overlook non-exclusive contests, and hence they cannot compare exclusive and non-exclusive contests. Yet, our results confirm the significance of non-exclusive contests for innovative settings. Second, while these papers assume that an organizer is interested in all solutions, we assume that an organizer is interested in the best solution—an objective more typical of innovation settings (cf. Terwiesch and Xu 2008). Third, while the above papers consider the impact of the agent's effort, we consider the impact of both the agent's effort and output uncertainty on her solution quality and hence on an organizer's profit. It is well established in the literature that uncertainty plays a prominent role in innovation contests in practice (cf. Boudreau et al. 2011). These aspects of our paper contribute to the innovation-contest theory and help us generate managerial insights.

2. The Model

Consider M innovation contests where M contest organizers ("he") elicit solutions to innovation-related problems from a set of N agents ("she"). In what follows, we describe our model of agents and organizers, and then present the equilibrium.

Agents. Each agent $i \in \{1, 2, ..., N\}$ develops a solution for each contest $m \in \{1, 2, ..., M\}$ she participates in, and generates an output $y_{im} \subseteq \mathbb{R} \cup \{-\infty, \infty\}$. The output y_{im} represents the quality of agent i's solution at contest m or its monetary value to organizer m. The output y_{im} is determined by agent i's effort e_{im} at contest m and agent i's output shock $\widetilde{\xi}_{im}$ at contest m, and it takes the following additive form: $y_{im} = y(e_{im}, \widetilde{\xi}_{im}) = r(e_{im}) + \widetilde{\xi}_{im}$. We next elaborate on these two terms.

First, each agent i can improve her output by exerting effort $e_{im} \subseteq \mathbb{R}_+$ at contest m. An agent's effort may represent the set of actions she takes to improve her output, such as "conducting a thorough patent search and literature review, or implementing rigorous quality control systems with high standards" (Terwiesch and Xu 2008, page 1532). For example, a logo designer may exert effort by drawing multiple sketches until she chooses the best one to submit (Ales et al. 2017c). The effort e_{im} leads to a deterministic improvement $r(e_{im})$ of the output, where r is an increasing and concave function of e_{im}, and r' is homogeneous of degree $-k$, where $k \geq 0$. This mild assumption is satisfied by functional forms that are commonly used in the literature such as linear and logarithmic forms. (We assume that all functions in the paper are thrice continuously differentiable.) Each agent has a capacity $\overline{E}$ on her total effort due to limited resources such as time and money.

Second, each agent faces uncertainty while developing her solution, which we capture with an output shock. As is standard in the literature, we assume that output shocks are independent and identically distributed.[7] The output shock $\widetilde{\xi}_{im}$ ($\in \Xi$) is independent for each agent i and for each contest m, and it follows a cumulative distribution function H and a density function h with $E[\widetilde{\xi}_{im}] = 0$ over support $\Xi = [\underline{s}, \overline{s}]$, where $\underline{s} < \overline{s}$, $\underline{s} \in \mathbb{R} \cup \{-\infty\}$, and $\overline{s} \in \mathbb{R} \cup \{\infty\}$. We assume that h is log-concave, i.e., $\log(h)$ is concave. This property is satisfied by most commonly used distributions such as Gumbel distribution used by Terwiesch and Xu (2008), uniform distribution used by Mihm and Schlapp (2017), and normal distribution. Throughout the paper, we analyze the impact of the agent's output uncertainty by changing the spread of the density h. To change the spread of a general log-concave density h, we use the notion of a scale transformation (e.g., Rothschild and Stiglitz 1978). When the output shock $\widetilde{\xi}_{im}$ is transformed by a scale transformation with parameter α, the transformed random variable $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ has a mean 0, and a variance $\alpha^2 Var(\widetilde{\xi}_{im})$. Thus, when $\alpha > 1$, the transformed density is more spread out. Let $\widetilde{\xi}^N_{(j)m}$ be a random variable that represents the j-th largest output shock among $\{\widetilde{\xi}_{1m}, \widetilde{\xi}_{2m}, ..., \widetilde{\xi}_{Nm}\}$, and let $\mu^N_{(j)} = E[\widetilde{\xi}^N_{(j)m}]$. Noting that $\widetilde{\xi}^N_{(j)m}$ is the $(N-j+1)$-st order statistic among N random variables, its density function is $h^N_{(j)} = \frac{N!}{(N-j)!(j-1)!} H(s)^{N-j}(1-H(s))^{j-1}h(s)$.

Agent i's utility $U_i = U(e_i, x_i) : \mathbb{R}^{2M}_+ \to \mathbb{R}$ is defined over the vector of efforts $e_i \equiv (e_{i1}, e_{i2}, ..., e_{iM})$ she exerts and the vector of awards $x_i \equiv (x_{i1}, x_{i2}, ..., x_{iM})$ she receives. Agent i's utility takes the form $U_i = \sum_{m=1}^M x_{im} - \psi(e_{i1}, e_{i2}, ..., e_{iM})$, and ψ represents the agent's disutility or cost associated with her effort. We assume that ψ has the following properties that seem consistent with the contest practice. First, each contest exhibits diseconomies of scale because an agent may have to allocate more time, effort, or money to improve her output at a certain contest. Thus, ψ is increasing in e_{im} with positive second partial derivatives; i.e., $\frac{\partial\psi}{\partial e_{im}} > 0$ and $\frac{\partial^2\psi}{\partial e^2_{im}} \geq 0$. This property is in line with the literature that assumes convex cost of effort in a single contest (e.g., Mihm and Schlapp 2017). Second, as discussed in §1, there is a potential economies of scope across contests because when an agent exerts more effort at one contest, the cost of her effort at another contest may decrease due to factors such as common learning (e.g., Willig 1979, Panzar and Willig 1981). For example, an agent who conducts a literature review for a contest at InnoCentive or Topcoder may find it less costly to conduct literature reviews for other contests of the same subject category. Thus, ψ has negative cross-partial derivatives; i.e., $\frac{\partial^2\psi}{\partial e_{il}\partial e_{im}} < 0$ for all $l \neq m$.

As the tractability of the general cost function ψ is limited, we assume the following form:

$$\psi(e_{i1}, e_{i2}, ..., e_{iM}) = \eta\left(\sum_{m=1}^{M} \phi(e_{im})\right), \tag{1}$$

where η is an increasing and homogeneous function of degree b (< 1), ϕ is an increasing and homogeneous function of degree p (> 1). We further assume that $bp \geq 1$ to ensure that $\eta \circ \phi$ is a convex function, and that either $bp > 1$ or $k > 0$ (where r' is homogeneous of degree $-k$). Lemma EC.4 of Online Appendix shows that ψ in (1) exhibits both diseconomies of scale and economies of scope as discussed above. Note that when there is a single contest (i.e., $M = 1$), ψ in (1) boils down to a convex cost function that subsumes the cost functions used in the literature, such as $\psi(e) = ce$ used by Terwiesch and Xu (2008), $\psi(e) = ce^{bp}$, where $bp \geq 1$ used by Ales et al. (2017b,c), and $\psi(e) = ce^2$ used by Mihm and Schlapp (2017).

Organizers. As is common in practice and in the literature discussed in §1, we assume a winner-take-all award scheme. Specifically, each organizer m gives an award A_m to the agent with the largest output, i.e., the winner at contest m. The winner-take-all award scheme is proven to be optimal in a contest where the output shock density h is log-concave as in our setting (see Proposition 3 of Ales et al. 2017c). Under the winner-take-all award scheme, if agent i wins contest m, her award is $x_{im} = A_m$; otherwise, $x_{im} = 0$. Consistent with the innovation-contest literature (Terwiesch and Xu 2008, Mihm and Schlapp 2017), we assume that each organizer is interested in the largest output in his contest. For example, in a logo-design contest, an organizer is interested in the quality of the best logo because he uses only the best logo. Thus, organizer m's profit Π_m consists of the largest output in his contest minus the award he gives, i.e., $\Pi_m = \max_i \; y_{im} - A_m$.

The sequence of events is as follows. First, awards $(A_1, A_2, ..., A_M)$ of all contests are announced, and then each agent i determines her effort e_{im} at each contest m she participates in, considering her total cost of effort ψ. Then, each agent i observes her output shock $\widetilde{\xi}_{im}$, and generates an output y_{im} at each contest m. Finally, each organizer m collects solutions from agents who participate in contest m, and gives the award A_m to the winner.

Equilibrium among agents. We next define and characterize Nash equilibrium of the subgame among agents. As is common in the innovation-contest literature, we focus on symmetric pure-strategy Nash equilibrium (hereafter, symmetric equilibrium), and denote each agent's equilibrium effort at contest m by e_m^*. To solve for equilibrium, we first derive agent i's probability of winning contest m by exerting effort e_{im} given that all other agents exert effort e_m^* at contest m:

$$P_m(e_{im}, e_m^*) = \int_{s\in\Xi} H(s + r(e_{im}) - r(e_m^*))^{N-1} h(s) ds. \tag{2}$$

Agent i chooses her effort e_{im} at each contest m to maximize her expected utility subject to a capacity constraint by solving the following problem:

$$\max_{(e_{i1}, e_{i2}, ..., e_{iM})} \sum_{m=1}^{M} A_m P_m(e_{im}, e_m^*) - \psi\left(e_{i1}, e_{i2}, ..., e_{iM}\right) \text{ s.t. } \sum_{m=1}^{M} e_{im} \leq \overline{E}. \tag{3}$$

In a symmetric equilibrium, each agent exerts effort e_m^* at contest m that solves (3), and we show the existence and uniqueness of e_m^* in §EC.1 of Online Appendix. In Lemma EC.1 of Online Appendix, we present sufficient conditions for the concavity of the agent's problem, and for the rest of the paper, we assume that these conditions are satisfied. Similar conditions are commonly used in the literature (e.g., Terwiesch and Xu 2008, Ales et al. 2017c, Mihm and Schlapp 2017).

We next characterize the unique symmetric equilibrium among agents. Note that all proofs are presented in Appendix.

LEMMA 1. *Let $\widehat{e}_m$ $(m \in \{1, 2, ..., M\})$ be the solution to the following set of equations:*

$$A_m r'(\widehat{e}_m) I_N = \eta' \left(\sum_{l=1}^{M} \phi(\widehat{e}_l) \right) \phi'(\widehat{e}_m) \text{ for all } m \in \{1, 2, ..., M\}, \text{ where } I_N \equiv \int_{s\in\Xi} (N-1) H(s)^{N-2} h(s)^2 ds. \tag{4}$$

When $\sum_{m=1}^{M}\widehat{e}_m<\overline{E}$, the unique symmetric-equilibrium effort at contest m is $e_m^*=\widehat{e}_m$, and when $\sum_{m=1}^{M}\widehat{e}_m\geq\overline{E}$, the unique symmetric-equilibrium effort e_m^* at contest m ($\in\{1,2,...,M\}$) satisfies:

$$A_m r'(e_m^*)I_N-\eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_m^*)=A_1 r'(e_1^*)I_N-\eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_1^*)\ \text{and}\ \sum_{m=1}^{M}e_m^*=\overline{E}. \quad (5)$$

Lemma 1 shows that when the agent's capacity constraint is not binding, each agent chooses her effort by optimally balancing the marginal impact of her effort on her expected award from each contest with the marginal impact of her effort on her total cost. When the agent's capacity constraint is binding, the agent optimally allocates her total effort among contests. Note that an agent's equilibrium efforts at all contests are interlinked via the common cost function ψ or via the capacity $\overline{E}$ on the agent's total effort. While our main model considers no fixed cost of participation and assumes that each agent participates in M contests, in §3.3, we incorporate a fixed cost of participation and consider the case where each agent participates in a limited number of contests.

Coordinator. In our main analysis, we assume that a *coordinator* determines the awards at all contests. This assumption is consistent with practice for two reasons. First, as discussed in §1, many organizations such as Elanco and Gates Foundation run multiple contests in parallel, and such an organization determines the awards at all of its contests. Second, as we discuss in §1, our interviews with practitioners at InnoCentive and Topcoder reveal that such a platform acts as a coordinator either by determining all awards on behalf of its customers or by instructing its customers in setting awards. We assume that the coordinator aims to maximize the expected average profit of organizers (hereafter, average profit) which is given by $\overline{\Pi}\equiv(1/M)(E[\sum_{m=1}^{M}\max_i\ y_{im}]-\sum_{m=1}^{M}A_m)$. Given the equilibrium effort e_m^*, we have $\max_i\ y_{im}=\max_i\ \{r(e_m^*)+\widetilde{\xi}_{im}\}=r(e_m^*)+\max_i\ \widetilde{\xi}_{im}=r(e_m^*)+\widetilde{\xi}_{(1)m}^N$. Thus, the coordinator's objective is to maximize the average profit

$$\overline{\Pi}=\frac{\sum_{m=1}^{M}r(e_m^*)}{M}+\frac{E\left[\sum_{m=1}^{M}\widetilde{\xi}_{(1)m}^N\right]}{M}-\frac{\sum_{m=1}^{M}A_m}{M}. \quad (6)$$

The objective function in (6) seems to coincide with the objective of a platform because a platform aims to increase value created for each customer, and this value is captured by an organizer's profit in our model. The objective function in (6) also seems suitable for an organization such as Elanco or Gates Foundation when it determines whether to run contests in parallel.[8] On the other hand, when an organization such as Elanco or Gates Foundation determines whether to run a new contest in parallel with others or to never run it (and hence lose the potential profit), a more suitable objective may be to maximize the total profit $\Pi^{\Sigma}=\sum_{m=1}^{M}\Pi_m$ from contests. We analyze this alternative objective in §4.1.

To characterize the optimal set of awards, we introduce two functions $g(x)=((\eta\circ\phi)'/r')^{-1}(x)$ and $\varphi(x)=(r'/\phi')(x)$, and we make two assumptions (similar assumptions are common in the literature reviewed in §1). First, we assume that $r'(g(x))g'(x)$ is decreasing in x (which holds if and only if $2-2k-bp<0$) so that organizer m's profit Π_m is concave in his award A_m. This assumption along with the capacity on the agent's total effort ensures that the coordinator always sets finite awards. Second, we assume that the agent's problem is sufficiently concave (e.g., when $b>0$ and $k\geq 1$ or $k>0$ and b is close to 1) so that agents face sufficiently large diminishing marginal returns compared to the cost they incur. Note that all assumptions we make on the effort function r and the cost function ψ are satisfied by effort and cost functions that are commonly used in the literature that focuses on a single contest. For example, our assumptions hold under the Terwiesch and Xu (2008) model where $r(e)=\theta\log(e)$, $\psi(e)=ce$, and $\theta,c>0$; under the Ales et al. (2017b,c) model where $r(e)=\theta(e^{1-a}-1)/(1-a)$, $\psi(e)=ce^{pb}$, $a\geq 1$, $pb\geq 1$, and $b\in(0,1)$; and under the Mihm and Schlapp (2017) model where $r(e)=\theta e$, $\psi(e)=ce^{pb}$, $\theta,c>0$, $pb=2$, and b is sufficiently close to 1. The following lemma characterizes the optimal set of awards.

Lemma 2. *Let* $\Phi(A)=r'(e^*)\,g'\left(AI_NM^{1-b}\right)I_NM^{1-b}-1$ *and* $\overline{A}=M^{b-1}g^{-1}\left(\overline{E}/M\right)/I_N$. *If* $\Phi(\overline{A})\geq 0$, *then* $\overline{\Pi}$ *is maximized at* $A_m^*=A^*=\overline{A}$ *and* $e_m^*=e^*=\overline{E}/M$. *If* $\Phi(\overline{A})<0$, *then there exists a unique* $\widehat{A}$ *such that* $\Phi(\widehat{A})=0$, *and* $\overline{\Pi}$ *is maximized at* $A_m^*=A^*=\widehat{A}$ *and* $e_m^*=e^*=g(A^*I_NM^{1-b})$.

Lemma 2 shows that the average profit $\overline{\Pi}$ is maximized when the award at each contest is A^*. This is because agents face diminishing marginal returns in their efforts, so identical awards improve the average of best outputs across all contests $\left(\text{i.e., } \frac{1}{M}\sum_{m=1}^{M}(r(e_m^*)+\mu_{(1)}^N)\right)$, and hence improve $\overline{\Pi}$. Let Π^* be an organizer's profit when the award at each contest is A^*. Under the optimal award A^* in Lemma 2, the average profit can be written as:

$$\overline{\Pi}=\Pi^*=r(e^*)+\mu_{(1)}^N-A^*. \tag{7}$$

Lemma 2 further shows that the optimal award A^* depends on whether the agent's capacity constraint is binding. When the agent's capacity constraint is not binding, it is optimal for the coordinator to set the awards to balance the marginal benefit and the marginal cost of an award on the average profit. However, when the agent's capacity constraint is binding, it is optimal for the coordinator to set the awards at $\overline{A}$, which is just enough to induce each agent to exert a total effort $\overline{E}$, because a larger award cannot improve an agent's effort beyond her capacity $\overline{E}$.

3. Main Analysis

This section proceeds as follows. In §3.1, we compare exclusive and non-exclusive contests. In §3.2, we analyze how an organizer's profit changes with the number of contests. In §3.3, we enrich our analysis by first incorporating a fixed cost of participation, and then by considering each agent's participation in a limited number of contests. In §3.4, we discuss our managerial insights.

3.1. Exclusive versus Non-Exclusive Contests

In this section, we analyze when agents should be discouraged from participating in multiple contests. In practice, an organization such as Gates Foundation or a platform such as Topcoder can discourage agents from participating in multiple contests, for example, by allowing submission only to a single contest. We refer to the case where each agent can participate in only one contest as the exclusive case, and the case where each agent can participate in multiple contests as the non-exclusive case. Note that in our model and in Lemmas 1 and 2, $M \geq 1$ characterizes equilibrium and optimal awards under M non-exclusive contests, and $M = 1$ characterizes equilibrium and optimal awards under a single exclusive contest. We next compare exclusive and non-exclusive cases.[9]

Theorem 1. *Let $\overline{\Pi}^X$ be the average profit when the coordinator optimally allocates agents and awards in the exclusive case. Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, there exists α_0 such that the average profit in the non-exclusive case $\overline{\Pi}$ is greater than that in the exclusive case $\overline{\Pi}^X$ for any $\alpha > \alpha_0$.*

Theorem 1 shows that when the agent's output uncertainty is sufficiently large, the non-exclusive case yields a larger average profit than the exclusive case; see Figure 1. To generate further insights, we use the following effort and cost functions that subsume the effort and cost functions that are commonly used in the literature (e.g., Terwiesch and Xu 2008, Körpeoğlu and Cho 2017).

Assumption 1. *$r(e) = \theta\log(e)$, $\eta(e) = ce^b$, and $\phi(e) = e^p$, where $\theta, c > 0$, $b \in (0,1)$, and $p \geq 1/b$.*

The following corollary shows that Theorem 1 is not an asymptotic result, and it characterizes α_0.

Corollary 1. *Consider two exclusive contests with N_1 and N_2 agents, and let $\overline{\Pi}^X$ be the average profit in this case. Suppose that Assumption 1 holds, and that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Let $\alpha_1 \equiv \frac{\theta^2 I_{N_1}}{p^2 b^2 \overline{E}^{bp}} \max\{I_{N_1}, I_{N_2}, 2^{1+b(p-1)} I_{N_1+N_2}\}$ and $\alpha_2 \equiv \frac{\theta}{bp} \frac{\log\left(I_{N_1} I_{N_2}\right) - 2\log\left(2^{1-b} I_{N_1+N_2}\right)}{2\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_1} - \mu_{(1)}^{N_2}}$. Then, the average profit in the non-exclusive case $\overline{\Pi}$ is greater than that in the exclusive case $\overline{\Pi}^X$ if $\alpha \geq \alpha_0 \equiv \max\{\alpha_1, \alpha_2\}$. Moreover, when $\overline{E}$ is sufficiently large, $\overline{\Pi}$ is greater than $\overline{\Pi}^X$ if and only if $\alpha \geq \alpha_0 = \alpha_2$.*

We next discuss the intuition of Theorem 1 using the setting in Assumption 1. The average profit $\overline{\Pi}$ depends on the effort term $r(e^*)$, the shock term $\widehat{\mu}_{(1)}^N$ $(= E[\widehat{\xi}_{(1)m}^N] = E[\alpha\widetilde{\xi}_{(1)m}^N])$, and the award term A^*. Figure 1 compares these three terms and the average profit in exclusive and non-exclusive cases as a function of the agent's output uncertainty measured by α. Because the award term is the same in both cases, whether the average profit is larger in the exclusive or non-exclusive case depends on effort and shock terms. On one hand, the shock term $\widehat{\mu}_{(1)}^N$ in the non-exclusive case is greater than that in the exclusive case because a non-exclusive contest attracts a larger number of

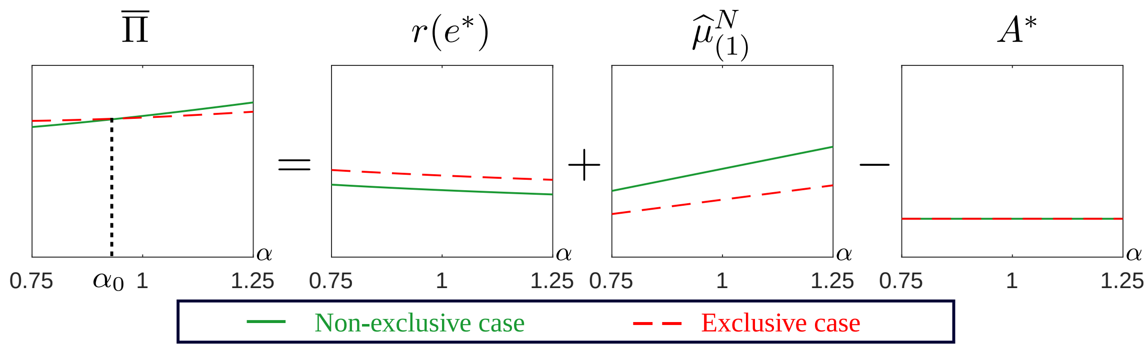


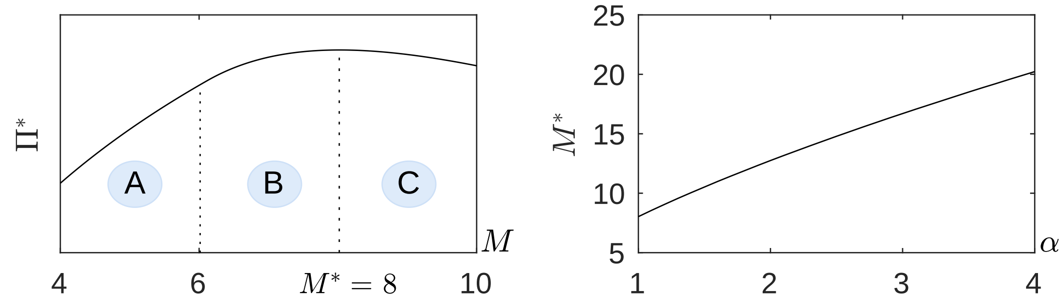
Figure 1 The average profit $\overline{\Pi}$ and effort, shock, and award terms, respectively, in exclusive and non-exclusive cases as a function of the scale parameter α. Setting: $\widetilde{\xi}_{im} \sim$ Gumbel with mean 0 and scale parameter 1, $M = 5$, $N = 100$, $\overline{E} = 5$, $r(e) = 2\log(e)$, $\eta(e) = 0.1e^{0.9}$, and $\phi(e) = e^2$.

agents, and hence benefits from a more diverse set of solutions. On the other hand, the effort term $r(e^*)$ in the exclusive case is larger than that in the non-exclusive case. This is because a smaller number of agents compete in an exclusive contest, so each agent exerts more effort in an exclusive contest. As the agent's output uncertainty measured by α increases, the difference between effort terms in exclusive and non-exclusive cases stays the same, but the difference between shock terms increases. Thus, when α is above a threshold α_0, the difference between shock terms dominates the difference between effort terms, so the average profit in the non-exclusive case is larger than that in the exclusive case.[10]

Theorem 1 and Corollary 1 have important implications for the contest theory and practice. First, these results suggest that in practice, organizers benefit from non-exclusive contests when they seek innovative solutions rather than low-novelty solutions (cf. Terwiesch and Xu 2008).[11] For example, InnoCentive may maximize the outcome of theoretical challenges that seek innovative solutions by encouraging agents to participate in multiple contests. In contrast, Topcoder may maximize the outcome of development challenges that seek low-novelty solutions by discouraging agents from participating in more than one of these contests (e.g., restricting submissions similarly to Gates Foundation). Second, although the prior research assumes exclusive contests, it is not only common for agents to participate in multiple contests in practice (see discussions in §1), but also often beneficial to organizers, as Theorem 1 and Corollary 1 show. Thus, although assuming exclusive contests may be reasonable for the specific examples the prior literature considers, relaxing this assumption is essential for studying multiple innovation contests. Therefore, in the following section, we analyze multiple non-exclusive contests while addressing exclusive contests in §3.3.

3.2. Optimal Number of Contests

In this section, we assume non-exclusive contests and analyze how the average profit $\overline{\Pi}$ changes with the number of contests M.



(a) Π^* as a function of M when $\alpha = 1$. (b) M^* as a function of α.

Figure 2 (a) M values where there is no scarcity effect (region A), there is a scarcity effect but it is dominated by the scope effect (region B), and the scope effect is dominated by the scarcity effect (region C); and (b) how the optimal number of contests M^* changes with the scale parameter α. Setting: $\widetilde{\xi}_{im} \sim$ Gumbel with mean 0 and scale parameter 1, $N = 100$, $\overline{E} = 1.5$, $r(e) = \log(e)$, $\eta(e) = 0.1e^{0.5}$, and $\phi(e) = e^2$.

Theorem 2. *The average profit $\overline{\Pi}$ is unimodal in the number of contests M, i.e., there exists $M^* \in [1, \infty)$ such that $\frac{\partial \overline{\Pi}}{\partial M} > 0$ for all $M < M^*$ and $\frac{\partial \overline{\Pi}}{\partial M} < 0$ for all $M > M^*$.*

Theorem 2 shows that there is an optimal number of contests M^* that maximizes the average profit. This is because each organizer's profit Π^* is maximized when there are M^* contests; see Figure 2(a). The intuition is as follows. Until the agent's capacity constraint binds, her total effort increases with the number of contests M because it is optimal for the agent to participate in more contests (see the proof of Theorem 2). Increasing the number of contests M has two effects. First, when an agent's capacity constraint binds, she splits her total effort among more contests, and hence exerts less effort at each contest. This "scarcity effect" reduces each organizer's profit Π^*. Second, as M increases, each agent enjoys a larger economies of scope, and the coordinator can utilize this larger economies of scope by reducing the award A^* at each contest. This "scope effect" improves each organizer's profit Π^*. When the number of contests is small (see region A in Figure 2(a) where $M \leq 6$), the agent's capacity constraint does not bind, so there is no scarcity effect. Hence, the scope effect leads to a larger profit for each organizer. When the number of contests is large (see regions B and C in Figure 2(a) where $M > 6$), the agent's capacity constraint binds, so the scarcity effect is positive. However, the benefit from the scope effect mitigates the reduced effort due to the scarcity effect, so each organizer's profit increases up to the optimal number of contests M^* (see region B in Figure 2(a)). Yet, as the number of contests gets sufficiently large, the benefit from the scope effect no longer mitigates the reduced effort due to the scarcity effect, so each organizer's profit decreases (see region C in Figure 2(a)). Thus, each organizer's profit Π^* is unimodal in the number of contests M, and there is an optimal number of contests M^*. This result suggests that an organization such as Elanco or Gates Foundation may benefit from running multiple contests that exhibit economies of scope (e.g., due to common learning), but only up to the optimal number of contests M^*. The following corollary shows, interestingly, that M^* increases with the agent's output uncertainty.

COROLLARY 2. *Suppose that the optimal number of contests $M^* > 1$, and the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, M^* is increasing in α.*

Corollary 2 shows that the optimal number of contests M^* is closely related to the spread of the output shock $\widetilde{\xi}_{im}$. Specifically, as Figure 2(b) illustrates, when the spread of the output shock $\widetilde{\xi}_{im}$ increases via a scale transformation with $\alpha > 1$, the optimal number of contests M^* increases. The intuition is as follows. As the agent's output uncertainty increases, the marginal impact of the agent's effort on her expected total award decreases, so the agent tends to reduce her effort. Less effort leads to a smaller scarcity effect and a smaller scope effect. Yet, as we show in Corollary 2, the scarcity effect decreases with the agent's output uncertainty more than the scope effect, and hence the scope effect outweighs the scarcity effect up to a larger number of contests M^*. This finding suggests that organizers benefit from a larger number of contests when they seek innovative solutions rather than low-novelty solutions.

3.3. Fixed Cost of Participation and Participation in a Limited Number of Contests

In this section, we enrich our analysis by first incorporating a fixed cost of participation and then considering a case where each agent participates in a limited number of contests.

Fixed cost of participation. We first consider a case where each agent incurs a fixed cost c_f for each contest she participates in, and analyze the impact of the fixed cost on the agent's participation in multiple contests. As setting equal awards for all contests is optimal (see Lemma EC.5 of Online Appendix), we assume that the award of each contest is A. To isolate the impact of the fixed cost, we relax the agent's capacity constraint, so her utility from participating in M contests is

$$U[M] = \frac{AM}{N} - M^b \eta(\phi(e^*)) - Mc_f, \tag{8}$$

where e^* is the equilibrium effort as given in Lemma 1. If the agent's participation condition holds (i.e., $U[M] \geq 0$), the agent finds it beneficial to participate in M contests.[12] We assume that the fixed cost c_f is not prohibitively high so that under award A, each agent participates in at least one contest (i.e., $U[1] \geq 0$). The following proposition characterizes the relationship between the agent's participation and the number of contests M.

PROPOSITION 1. *(a) Suppose $k \geq 1$. The agent's participation condition holds for any M.*
(b) Suppose $k < 1$, and that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, there exists a unique $\overline{M}$ such that the agent's participation condition is violated when $M > \overline{M}$. Also, $\overline{M}$ is increasing in α.

Proposition 1(a) shows, interestingly, that the presence of a fixed cost does not necessarily limit the number of contests that each agent participates in. The intuition is as follows. The agent's participation condition (i.e., $U[M] \geq 0$) depends on the agent's utility $U[M]$; and the number of contests M has two opposing effects on $U[M]$. On one hand, as M increases, the agent can improve her expected total award by participating in more contests, and this raises the agent's utility $U[M]$. On the other hand, each agent increases her effort e^* to compete in more contests and to benefit from economies of scope, and this reduces $U[M]$. Depending on which effect dominates, the agent's utility can increase or decrease. When $k \geq 1$ (where r' is homogeneous of degree $-k$), the marginal impact of the agent's effort on her output decreases quickly, so as M increases, each agent does not increase her total effort significantly, leading to a small increase in her cost of effort. The increased expected total award dominates the increased cost of effort, so the agent's utility increases with M (see the proof of Proposition 1). Thus, the agent's participation condition holds for any M.

Proposition 1(b) shows that when $k < 1$, the agent's participation condition holds for a limited number of contests. The intuition is as follows. When $k < 1$, the marginal impact of the agent's effort on her output decreases slowly, so as M increases, each agent increases her total effort significantly, leading to a substantial increase in her cost of effort. The increased cost of effort dominates the increased expected total award, eventually leading her utility to decrease. Thus, when an agent participates in more than $\overline{M}$ contests, her participation condition is violated. Proposition 1(b) further shows that $\overline{M}$ increases with the agent's output uncertainty. This result is in line with Corollary 2, which shows that the optimal number of contests M^* increases with the agent's output uncertainty. Thus, these results suggest that both organizers and agents benefit from a larger number of contests when organizers seek innovative solutions rather than low-novelty solutions.

Agent's participation in a limited number of contests. In practice, an agent can participate in a limited number of contests, either because these contests are exclusive as in §3.1 or because the agent's participation condition prevents her from entering all contests (even though these contests are non-exclusive) as discussed above.

For tractability, we consider a setting with N agents where each agent enters a single contest. We compare the average profit when N agents enter a single contest with the average profit when N_1 agents enter one contest and $N_2(= N - N_1)$ agents enter the other contest. To isolate the impact of an agent's participation in a limited number of contests, we relax the agent's capacity constraint.

PROPOSITION 2. *Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Under Assumption 1, two contests with N_1 and N_2 agents yield a larger average profit $\overline{\Pi}^L$ than a single contest with $N_1 + N_2$ agents if and only if $\alpha < \alpha_L \equiv \frac{\theta}{bp}\frac{\log\left(I_{N_1} I_{N_2}\right) - 2\log\left(I_{N_1+N_2}\right)}{2\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_1} - \mu_{(1)}^{N_2}}$.*

Proposition 2 shows that when each agent participates in a limited number of contests, the average profit $\overline{\Pi}^L$ increases with more contests if and only if the agent's output uncertainty is sufficiently small. This is because each organizer's profit $\Pi^{*,L}$ increases with the number of contests M if and only if the agent's output uncertainty is sufficiently small. The intuition is as follows. Let N_m be the number of agents at contest m. When each agent participates in a subset of contests, as M increases, agents are split among more contests, so the number of agents N_m at each contest m decreases. At contest m, this decrease in N_m can affect the organizer's profit $\Pi^{*,L} = r(e^*) + \widehat{\mu}_{(1)}^{N_m} - A^*$ through the effort term $r(e^*)$, the shock term $\widehat{\mu}_{(1)}^{N_m}$, and the award term A^*. First, the award term $A^* = \theta/(bp)$ in the setting of Proposition 2, so A^* does not change with N_m. Second, as N_m decreases, fewer agents compete at contest m, and the impact of each agent's effort on her expected total award is generally larger, so each agent at contest m generally exerts more effort.[13] Thus, the effort term $r(e^*)$ generally increases as N_m decreases. Third, as N_m decreases, organizer m receives a less diverse set of solutions; i.e., the shock term $\widehat{\mu}_{(1)}^{N_m}$ decreases. When the agent's output uncertainty is small, the increase in the effort term $r(e^*)$ outweighs the decrease in the shock term $\widehat{\mu}_{(1)}^{N_m}$, so each organizer's profit $\Pi^{*,L}$ increases with more contests. In contrast, when the agent's output uncertainty is large, the decrease in the shock term outweighs the increase in the effort term, so each organizer's profit decreases with more contests.

3.4. Managerial Insights

In this section, we discuss the key managerial insights that stem from our results. We classify our managerial insights based on the agent's output uncertainty, and summarize them in Table 1.

When the agent's output uncertainty is small, Theorem 1 and Corollary 1 show that each organizer's profit is maximized if agents are discouraged from participating in multiple contests; i.e., exclusive contests are optimal. Proposition 2 builds on this result, and shows that it is optimal to run multiple exclusive contests where each agent participates in a single contest. Thus, we advise practitioners who seek low-novelty solutions to run multiple contests in parallel, yet discourage agents from participating in multiple contests. This managerial insight seems consistent with practice. For instance, as discussed in §1, Topcoder organizes multiple parallel development challenges that seek low-novelty solutions but aims to focus each agent's effort on a single such contest.

When the agent's output uncertainty is large, Theorem 1 and Corollary 1 show that each organizer's profit is maximized if agents are encouraged to participate in multiple contests; i.e., non-exclusive contests are optimal. Theorem 2 builds on this result, and shows that each organizer's profit increases with the number of contests M only up to an optimal number of contests M^*. Consistently, Proposition 1(b) together with Proposition 2 suggest that each organizer's profit decreases as M exceeds the threshold $\overline{M}$ over which the agent's participation condition is violated. These

Table 1 The summary of key results and managerial insights.

	Small uncertainty (e.g., when seeking low-novelty solutions)	Large uncertainty (e.g., when seeking innovative solutions)
Exclusive vs non-exclusive contests	Exclusive contests are optimal (Theorem 1 and Corollary 1).	Non-exclusive contests are optimal (Theorem 1 and Corollary 1).
Multiple contests or not	Running more contests than what each agent can participate in improves each organizer's profit (Proposition 2).	Each organizer's profit increases with the number of contests up to an optimal number of contests (Theorem 2).
Managerial insights	Run multiple contests in parallel (up to a certain number) but discourage agents from participating in multiple contests.	Run multiple contests in parallel (up to a certain number) and encourage agents to participate in multiple contests.

results together show that each organizer's profit increases with M only up to $\min\{M^*, \overline{M}\}$. Interestingly, Corollary 2 and Proposition 1(b) show that $\min\{M^*, \overline{M}\}$ increases with the agent's output uncertainty. Combining all these findings, we advise practitioners who seek innovative solutions to run multiple parallel contests up to a certain threshold, and to encourage agents to participate in multiple contests. This managerial insight seems to be consistent with practice. For instance, as discussed in §1, InnoCentive organizes multiple parallel theoretical challenges that seek innovative solutions, and agents are encouraged to participate in multiple such contests.

4. Extensions

In this section, we extend our main results to various cases. In §4.1, we consider the total profit of organizers (instead of the average profit) as the coordinator's objective. In §4.2, we consider the decentralized case where each organizer sets the award at his contest and competes for agents' efforts. In §4.3, we consider an alternative way of modeling economies of scope.

4.1. Alternative Objective for Coordinator

Our main model in §2 assumes that the coordinator maximizes the average profit. As discussed in §2, this objective seems to be aligned with the objective of a contest platform, and with the objective of an organization such as Elanco or Gates Foundation when it determines whether to run contests in parallel. In this section, we analyze the case where the coordinator maximizes the total profit of organizers (hereafter, total profit). This alternative objective for the coordinator complements the one in §2, and provides insights for an organization that considers whether to run a new contest in parallel with others or to never run it (and hence lose the potential profit). We first discuss when the coordinator should run exclusive contests.

Corollary 3. *Theorem 1 holds when the coordinator maximizes the total profit.*

Corollary 3 shows that when the agent's output uncertainty is sufficiently large, the non-exclusive case yields a larger total profit than the exclusive case, so agents should be encouraged to participate in multiple contests. Corollary 3 has exactly the same intuition as Theorem 1.

We next discuss the optimal number of contests. Consistent with practice, we restrict attention to contests with non-zero awards. Before presenting the main result of this section, we make the following assumption.

Assumption 2. *When $M=1$, $\Pi^* = r(e^*) + \mu_{(1)}^N - A^* > 0$, where e^* and A^* are as in Lemma 2.*

Assumption 2 states that when there is a single contest (i.e., $M=1$), an organizer can make positive expected profit by giving the optimal award A^*. We make this mild assumption because otherwise, increasing the number of contests may add up negative profits. The following proposition extends Theorem 2 by showing that the coordinator's objective is unimodal in the number of contests.

Proposition 3. *Suppose that the coordinator sets non-zero awards at M contests. Under Assumption 2, $\Pi^{*,\Sigma} \equiv \sum_{m=1}^{M} \Pi^*$ is unimodal in M, i.e., there exists $M^{*,\Sigma}$ such that $\frac{\partial \Pi^{*,\Sigma}}{\partial M} > 0$ for all $M < M^{*,\Sigma}$ and $\frac{\partial \Pi^{*,\Sigma}}{\partial M} < 0$ for all $M > M^{*,\Sigma}$.*

Proposition 3 shows that even when the coordinator maximizes the total profit, there is a limit on the optimal number of contests. To explain the intuition, we first discuss how each organizer's profit Π^* changes with the number of contests M, and then discuss the impact of M on the total profit. When M increases, as discussed in §3.2, each organizer's profit Π^* increases as long as the scope effect outweighs the scarcity effect. Yet, when M is above a threshold M^*, the scarcity effect outweighs the scope effect, so each organizer's profit Π^* decreases with M. When the coordinator maximizes the total profit, even if each organizer's profit Π^* decreases with M, the total profit $M\Pi^*$ may still increase with M, and hence it may be optimal to run more contests than M^*. However, Proposition 3 shows that as M increases, the decrease in each organizer's profit due to the scarcity effect becomes so large that the total profit decreases as well. Thus, in line with Theorem 2, there is an optimal number of contests $M^{*,\Sigma}$ even when the coordinator maximizes the total profit.

4.2. Decentralized Contests

In this section, we consider the decentralized case where each organizer sets the award at his contest and competes for agents' efforts. Given that other organizers give the award $A_{j\neq m} = A^{*,D}$, and that each agent exerts the equilibrium effort e_m^* at contest m as in Lemma 1, each organizer m chooses his award A_m to maximize his expected profit by solving the following problem:

$$\max_{A_m} \; r(e_m^*) + \mu_{(1)}^N - A_m. \tag{9}$$

We refer to $A^{*,D}$ that solves (9) as the equilibrium award in the decentralized case. As in §2, we focus on symmetric pure-strategy Nash equilibria for both organizers and agents.

PROPOSITION 4. *In the decentralized case, under Assumption 1, the following results hold.*
(a) *Let $\overline{\Pi}^X$ be the average profit when the coordinator optimally allocates agents and awards in the exclusive case. Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, there exists α_0 such that the average profit in the non-exclusive decentralized case $\overline{\Pi}^D$ is greater than $\overline{\Pi}^X$ for any $\alpha > \alpha_0$.*
(b) *There exist $M_1^{*,D}$ and $M_2^{*,D}$ such that $\frac{\partial\overline{\Pi}^D}{\partial M} > 0$ for all $M < M_1^{*,D}$ and $\frac{\partial\overline{\Pi}^D}{\partial M} < 0$ for all $M > M_2^{*,D}$.*

Proposition 4(a) extends Theorem 1 to the decentralized case, and has the same intuition as Theorem 1. The average profit in the exclusive centralized case $\overline{\Pi}^X$ is an upper bound for the average profit in the exclusive decentralized case where each agent determines which contest(s) to participate in and her effort, while each organizer determines his award. We use the upper bound $\overline{\Pi}^X$ because in the exclusive decentralized case, a pure-strategy Nash equilibrium among organizers may not exist. Note that whenever the average profit in the non-exclusive decentralized case $\overline{\Pi}^D$ is larger than the average profit in the exclusive centralized case $\overline{\Pi}^X$, each organizer's profit in the non-exclusive decentralized case is larger than that in the exclusive decentralized case.

Proposition 4(b) extends Theorem 2 to the decentralized case, and has the same intuition as Theorem 2. The only difference is that Theorem 2 shows that the average profit is unimodal in the number of contests M with a peak M^*, yet Proposition 4(b) shows two thresholds $M_1^{*,D}$ and $M_2^{*,D}$ such that each organizer's profit increases with M when $M < M_1^{*,D}$ and decreases with M when $M > M_2^{*,D}$. Nevertheless, this result corroborates the insight of Theorem 2 that multiple contests are beneficial to organizers only up to the optimal number of contests.

4.3. Alternative Model for Economies of Scope

Consistent with the innovation-contest literature (e.g., Terwiesch and Xu 2008, Ales et al. 2017c), our main model in §2 interprets an agent's effort as the set of actions she takes to improve her output, such as conducting literature review. Alternatively, effort can be interpreted as deterministic improvement an agent makes to her solution quality (e.g., Moldovanu and Sela 2001). These two interpretations lead to modeling economies of scope through the agent's cost function ψ, and this is consistent with the traditional definition of economies of scope (e.g., Willig 1979, Panzar and Willig 1981). In this section, we consider a third interpretation of effort as the time an agent spends on a contest. To do so, we consider spillover in the agent's output function rather than economies of scope in the agent's cost function. Specifically, the time agent i spends on one contest may improve her output at another contest, so her output at contest m is $y_{im} = \theta\left(e_{im} + a\sum_{l\neq m} e_{il}\right) + \widetilde{\xi}_{im}$, where

$a \in (0,1)$.[14] This model builds on the Sutton (2001) model of output spillover. The innovation-contest literature that focuses on a single contest commonly uses this type of a linear effort function with a convex cost function (e.g., Mihm and Schlapp 2017, Hu and Wang 2017). Consistent with Sutton (2001) and the innovation-contest literature, we assume that agent i's total cost of effort is $\sum_{l=1}^{M} \phi(e_{il})$, where ϕ is an increasing, convex, and homogeneous function of degree p (> 2). The cost function ϕ may represent the agent's disutility from spending time on a contest. As in our main model, we assume that each agent has a capacity $\overline{E}$ on her total effort.

Proposition 5. ***(a)*** *Let* $\overline{\Pi}^X$ *be the average profit when the coordinator optimally allocates agents and awards in the exclusive case. Suppose that the output shock* $\widetilde{\xi}_{im}$ *is transformed to* $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ *with a scale parameter* $\alpha > 0$. *Then, there exists* α_0 *such that the average profit in the non-exclusive case* $\overline{\Pi}$ *is greater than that in the exclusive case* $\overline{\Pi}^X$ *for any* $\alpha > \alpha_0$.
(b) *The average profit* $\overline{\Pi}$ *is unimodal in the number of contests* M, *i.e., there exists* M^* *such that* $\frac{\partial \overline{\Pi}}{\partial M} > 0$ *for all* $M < M^*$ *and* $\frac{\partial \overline{\Pi}}{\partial M} < 0$ *for all* $M > M^*$.

Proposition 5 extends Theorems 1 and 2, and presents somewhat expected results as there is a strong correlation between the three interpretations of effort and between output spillover and economies of scope. Specifically, when an agent spends more time on a contest, the deterministic part of her output, i.e., $\theta\left(e_{im} + a\sum_{l\neq m} e_{il}\right)$, at another contest also improves. Thus, when an agent improves her output at one contest, it is less costly to improve her output at another contest, leading to economies of scope across contests. This strong correlation between different interpretations of effort explains why the results in Proposition 5 are analogous to the results in Theorems 1 and 2.

5. Conclusion

In recent years, contests have gained popularity as a tool to outsource innovation from independent agents. Each year, organizations such as Elanco and Gates Foundation and platforms such as InnoCentive and Topcoder run thousands of contests, providing agents with several problems to work on. This multiple-contest environment leads to tensions that do not arise in a single-contest environment. Specifically, agents may benefit from economies of scope by working on multiple contests, yet due to limited resources, they may have to split their efforts among multiple contests or even refrain from participating in some of these contests, potentially reducing organizers' profits. Moreover, discouraging agents from participating in multiple contests may focus agents' efforts but may hinder the diversity of solutions at each contest. These trade-offs pose two important questions for practitioners that the academic literature has yet to answer: when should agents be discouraged from participating in multiple contests and how does the number of contests affect an organizer's profit? In this paper, we take the first step towards answering these questions.

We analyze these questions by building a model of innovation contests, and our analysis yields the following results. First, we show that when the agent's output uncertainty is large, the non-exclusive case where each agent can enter multiple contests generates larger profits for organizers than the exclusive case where each agent can enter only one contest. In contrast, when the agent's output uncertainty is small, the exclusive case generates larger profits for organizers than the non-exclusive case. Second, we show that an organizer's profit can increase up to an optimal number of contests, and the drivers for the optimal number of contests depend on the agent's output uncertainty. Taken together, our results provide two managerial insights. First, practitioners who seek innovative solutions may run multiple parallel contests that exhibit economies of scope, and encourage agents to participate in multiple contests. Second, practitioners who seek low-novelty solutions may run multiple parallel contests but may discourage agents from participating in multiple contests.

In addition to providing managerial insights, we make several technical contributions to the innovation-contest theory. First, while the prior literature focuses on a single contest, we study multiple contests and the resulting multidimensional optimization problem for each agent who determines her effort at each contest by considering her total cost of effort. Second, while the prior literature assumes no fixed cost of participation and unbounded efforts for agents, we consider an agent's fixed cost of participation and capacity constraint. Third, we propose a cost function that captures both diseconomies of scale at each contest and economies of scope across contests. While these features require special technical attention, they bring our paper closer to practice.

Our model has the following limitations that can lead to new research opportunities. First, to focus on the impact of multiple parallel contests, we assume identical organizers, but it may be interesting to consider heterogeneous organizers. Second, as is common in the literature, we use a static model while analyzing the impact of multiple parallel contests (see §1 for a detailed discussion). Consequently, our model overlooks an organizer's decision of whether to run multiple contests in parallel or to run them sequentially. However, our model captures the key trade-off that may arise in a sequential setting. Specifically, running contests in parallel rather than sequentially may lead to larger economies of scope, but may also induce agents to split their efforts.[15] Nonetheless, one may consider how to dynamically schedule multiple contests, potentially considering the duration of each contest to maximize the average or total profit.

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Appendix. Proofs

Proof of Lemma 1. In Lemma EC.2 of Online Appendix, we show that there exists a unique $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$ that solves the following set of equations:

$$A_m r'(\widehat{e}_m) I_N = \eta'\left(\sum_{l=1}^{M}\phi(\widehat{e}_l)\right)\phi'(\widehat{e}_m) \text{ for all } m \in \{1,2,...,M\}, \text{ where } I_N \equiv \int_{s\in\Xi}(N-1)H(s)^{N-2}h(s)^2 ds.$$

Using η' is homogeneous of degree $(b-1)$ and letting $g = ((\eta\circ\phi)'/r')^{-1}$, we have

$$\frac{A_m r'(\widehat{e}_m) I_N}{\eta'\left(\phi(\widehat{e}_m)\right)\phi'(\widehat{e}_m)} = \frac{A_m I_N}{g^{-1}(\widehat{e}_m)} = \left(\frac{\phi(\widehat{e}_m)}{\sum_{l=1}^{M}\phi(\widehat{e}_l)}\right)^{1-b} \quad \text{for all } m \in \{1,2,...,M\}. \tag{10}$$

Let λ be the Lagrange multiplier for the agent's capacity constraint. As shown in Lemma EC.2 of Online Appendix, a symmetric equilibrium satisfies the following Kuhn-Tucker conditions:

$$A_m r'(e_m^*) I_N - \left(\frac{\phi(e_m^*)}{\sum_{l=1}^{M}\phi(e_l^*)}\right)^{1-b}\eta'\left(\phi(e_m^*)\right)\phi'(e_m^*) - \lambda^* = 0, \; m \in \{1,2,...,M\}. \tag{11}$$

$$\lambda^*\left(\overline{E}-\sum_{m=1}^{M}e_m^*\right)=0,\ \sum_{m=1}^{M}e_m^*\leq\overline{E},\text{ and }e_m^*,\lambda^*\geq 0,\ m\in\{1,2,...,M\}. \tag{12}$$

Case 1: $\sum_{m=1}^{M}\widehat{e}_m<\overline{E}$. Then, $(\widehat{e}_1,\widehat{e}_2,...,\widehat{e}_M,\widehat{\lambda})$, where $\widehat{\lambda}=0$ is a solution to (11)-(12). Suppose to the contrary that there exists another solution $(\overline{e}_1,\overline{e}_2,...,\overline{e}_M,\overline{\lambda})$ to (11)-(12). Then, because $(\widehat{e}_1,\widehat{e}_2,...,\widehat{e}_M)$ is the unique solution to (10), we should have $\overline{\lambda}>0$, which means $\sum_{m=1}^{M}\overline{e}_m=\overline{E}$ and

$$\frac{A_mI_N}{g^{-1}(\overline{e}_m)}>\left(\frac{\phi(\overline{e}_m)}{\sum_{l=1}^{M}\phi(\overline{e}_l)}\right)^{1-b}\quad\text{for all } m\in\{1,2,...,M\}. \tag{13}$$

As $\sum_{m=1}^{M}\widehat{e}_m<\overline{E}$, there should exist $m\in\{1,2,...,M\}$ such that $\overline{e}_m>\widehat{e}_m$. As g^{-1} is increasing, we need to have $\frac{\phi(\overline{e}_m)}{\sum_{l=1}^{M}\phi(\overline{e}_l)}<\frac{\phi(\widehat{e}_m)}{\sum_{l=1}^{M}\phi(\widehat{e}_l)}$, which requires that $\sum_{l=1}^{M}\phi(\overline{e}_l)>\sum_{l=1}^{M}\phi(\widehat{e}_l)$. As $\sum_{m=1}^{M}\frac{\phi(\overline{e}_m)}{\sum_{l=1}^{M}\phi(\overline{e}_l)}=\sum_{m=1}^{M}\frac{\phi(\widehat{e}_m)}{\sum_{l=1}^{M}\phi(\widehat{e}_l)}=1$, there should exist $j\in\{1,2,...,M\}$ such that $\frac{\phi(\overline{e}_j)}{\sum_{l=1}^{M}\phi(\overline{e}_l)}>\frac{\phi(\widehat{e}_j)}{\sum_{l=1}^{M}\phi(\widehat{e}_l)}$, which means that $\overline{e}_j>\widehat{e}_j$ as $\sum_{l=1}^{M}\phi(\overline{e}_l)>\sum_{l=1}^{M}\phi(\widehat{e}_l)$. However, as $\frac{\phi(\overline{e}_j)}{\sum_{l=1}^{M}\phi(\overline{e}_l)}>\frac{\phi(\widehat{e}_j)}{\sum_{l=1}^{M}\phi(\widehat{e}_l)}$ and the left-hand side of (10) is decreasing in $\widehat{e}_m$, we have $\overline{e}_j<\widehat{e}_j$ from (10). Thus, we have a contradiction, so the unique symmetric equilibrium effort at contest $m\in\{1,2,...,M\}$ is $e_m^*=\widehat{e}_m$.

Case 2: $\sum_{m=1}^{M}\widehat{e}_m\geq\overline{E}$. In this case, the unique candidate for the symmetric equilibrium effort e_m^* at contest $m\in\{1,2,...,M\}$ satisfies (11)-(12), and these conditions boil down to

$$A_mr'(e_m^*)I_N-\eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_m^*)=A_1r'(e_1^*)I_N-\eta'\left(\sum_{l=1}^{M}\phi(e_l^*)\right)\phi'(e_1^*)\text{ and }\sum_{m=1}^{M}e_m^*=\overline{E}.\ \blacksquare$$

Proof of Lemma 2. Because the coordinator optimally sets equal awards at all contests by Lemma EC.5 of Online Appendix, without loss of optimality, the coordinator's problem can be rewritten as follows (where A is the award given at each contest, and $\mu_{(j)}^N=E[\widetilde{\xi}_{(j)m}^N]$):

$$\max_A\ r(e^*)+\mu_{(1)}^N-A\text{ s.t. }e^*=\min\{g(AI_NM^{1-b}),\overline{E}/M\}. \tag{14}$$

Note that the coordinator never sets A such that $g(AI_NM^{1-b})>\overline{E}/M$ because otherwise, reducing A improves the objective function in (14). Thus, without loss of optimality, (14) can be written as

$$\max_A\ r(g(AI_NM^{1-b}))+\mu_{(1)}^N-A\text{ s.t. }Mg(AI_NM^{1-b})\leq\overline{E}. \tag{15}$$

Let $\Phi(A)=r'\left(g\left(AI_NM^{1-b}\right)\right)g'\left(AI_NM^{1-b}\right)I_NM^{1-b}-1$ and $\overline{A}=M^{b-1}g^{-1}\left(\overline{E}/M\right)/I_N$. Note that Φ is the first derivative of the objective function in (15) with respect to A. Suppose that $\Phi(\overline{A})\geq 0$. Because $r'(g(x))g'(x)$ is decreasing in x (as assumed in §2), the objective function in (15) is concave in A, and hence $\Phi(A)$ is decreasing in A. Because $A>\overline{A}$ violates the constraint in (15), and because Φ is decreasing, $A^*=\overline{A}$ solves (15). Thus, $A_m=A^*=\overline{A}$ maximizes the average profit $\overline{\Pi}$, and $e_m^*=e^*=\overline{E}/M$ is the corresponding equilibrium effort. Suppose that $\Phi(\overline{A})<0$. Then, because $\lim_{x\to 0}r'(g(x))g'(x)=\infty$, $\Phi(0)>0$, which follows from $(r'\circ g)g'$ being homogeneous of degree $\frac{2-2k-bp}{bp+k-1}<0$, by the Intermediate Value Theorem, there exists $\widehat{A}$ such that $\Phi(\widehat{A})=0$. Note that $\widehat{A}$ is unique because Φ is decreasing. In this case, $A^*=\widehat{A}$ solves (15). Thus, $A_m=A^*=\widehat{A}$ maximizes the average profit $\overline{\Pi}$, and $e_m^*=e^*=g(A^*I_NM^{1-b})$ is the equilibrium effort. $\blacksquare$

Proof of Theorem 1. We prove the result for two contests but this result can be generalized to any number of contests $M>2$. We compare the average profit in exclusive and non-exclusive cases. In the exclusive case, let $N_m^{*,X}$ be the optimal number of agents and $A_m^{*,X}$ be the optimal award at contest $m\in\{1,2\}$. Let $e_m^{*,X}$ be the corresponding equilibrium effort at contest $m\in\{1,2\}$. Note that it is never optimal for the coordinator to set awards such that $e_m^{*,X}>\overline{E}$ (for $m\in\{1,2\}$) because the coordinator can improve the average profit by reducing the award at contest $m\in\{1,2\}$. Thus, by Lemma 1, the equilibrium effort in the exclusive case is $e_m^{*,X}=g\left(A_m^{*,X}I_{N_m^{*,X}}\right)$ at contest $m\in\{1,2\}$. Without loss of generality, suppose that $e_1^{*,X}\le e_2^{*,X}$. After incorporating the optimal solution, the average profit in the exclusive case becomes

$$\overline{\Pi}^X=\frac{1}{2}\sum_{m=1}^{2}(r(e_m^{*,X})+\mu_{(1)}^{N_m^{*,X}}-A_m^{*,X}). \tag{16}$$

In the non-exclusive case, suppose that the coordinator offers an award A at each contest so that the equilibrium effort at each contest $m\in\{1,2\}$ is $e_m^*=(e_1^{*,X}+e_2^{*,X})/2$. Note that because $\sum_{m=1}^2 e_m^*=(e_1^{*,X}+e_2^{*,X})\le\overline{E}$ and g^{-1} is increasing, from (4), we know that A satisfies

$$A=\frac{2^{b-1}}{I_N}g^{-1}\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right)\le\frac{2^{b-1}}{I_N}g^{-1}(e_2^{*,X})=\frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{I_N}.$$

Then, the average profit in the non-exclusive case can be written as

$$\overline{\Pi}=r\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right)+\mu_{(1)}^N-A\ge\frac{1}{2}\sum_{m=1}^{2}r(e_m^{*,X})+\mu_{(1)}^N-\frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{I_N}. \tag{17}$$

Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha>0$. The difference between the average profit in non-exclusive and exclusive cases satisfies

$$\overline{\Pi}-\overline{\Pi}^X\ge\alpha\left(\mu_{(1)}^N-\frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}}+\frac{1}{2}\sum_{m=1}^{2}\frac{A_m^{*,X}}{\alpha}-\frac{2^{b-1}A_2^{*,X}I_{N_2^{*,X}}}{\alpha I_N}\right). \tag{18}$$

As Lemma EC.7 of Online Appendix shows, $\lim_{\alpha\to\infty}A_m^{*,X}/\alpha=0$ for each $m\in\{1,2\}$. Also, because $N>N_m^{*,X}$ for a sufficiently large α (as shown in Lemma EC.10 of Online Appendix), $\widetilde{\xi}_{(1)_m}^N$ first-order stochastically dominates $\widetilde{\xi}_{(1)m}^{N_m}$ for $m\in\{1,2\}$ (and not vice versa), so we have $\mu_{(1)}^N>\frac{1}{2}\sum_{m=1}^2\mu_{(1)}^{N_m^{*,X}}$ for $m\in\{1,2\}$. Thus, there exists α_0 such that for any $\alpha>\alpha_0$, we have $\overline{\Pi}-\overline{\Pi}^X>0$. ∎

Proof of Corollary 1. Consider the exclusive case where N_1 and N_2 agents participate in contest 1 and 2, respectively. In the non-exclusive case, all $N(=N_1+N_2)$ agents participate in both contests. Let $A^{*,M,N}=\min\{\frac{\theta}{bp},M^{b(1-p)-1}\frac{cbp}{\theta I_N}\left(\overline{E}\right)^{bp}\}$. In the non-exclusive case, by incorporating Assumption 1 to Lemmas 1 and 2, we obtain the equilibrium effort $e^*=\left(\frac{\theta A^*I_N2^{1-b}}{cpb}\right)^{\frac{1}{pb}}$, where $A^*=A^{*,2,N}$. Then, the average profit in the non-exclusive case is

$$\overline{\Pi}=r(e^*)+\mu_{(1)}^N-A^*=\frac{\theta}{pb}\log\left(\frac{A^{*,2,N}\theta I_N2^{1-b}}{cbp}\right)+\mu_{(1)}^N-A^{*,2,N}.$$

The average profit in the exclusive case with two contests is

$$\overline{\Pi}^X = \left[\frac{\theta}{bp}\log\left(\frac{\theta\sqrt{A^{*,1,N_1}I_{N_1}A^{*,1,N_2}I_{N_2}}}{cbp}\right) + \frac{\mu_{(1)}^{N_1}+\mu_{(1)}^{N_2}}{2} - \frac{A^{*,1,N_1}+A^{*,1,N_2}}{2}\right].$$

When $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with $\alpha > 0$, we have $A^{*,M,N} = \min\{\frac{\theta}{bp}, M^{b(1-p)-1}\frac{\alpha cbp}{\theta I_N}\left(\overline{E}\right)^{bp}\}$. Thus, when $\alpha \geq \alpha_1 \equiv \frac{\theta^2 I_{N_1}}{p^2b^2\overline{E}^{bp}}\max\{I_{N_1}, I_{N_2}, 2^{1+b(p-1)}I_{N_1+N_2}\}$, we have $A^{*,2,N} = A^{*,1,N_m} = \frac{\theta}{pb}$. The difference between the average profit in the non-exclusive and the exclusive case is

$$\overline{\Pi} - \overline{\Pi}^X = \frac{\theta}{2bp}\log\left(\frac{2^{2-2b}I_{N_1+N_2}^2}{I_{N_1}I_{N_2}}\right) + \alpha\mu_{(1)}^{N_1+N_2} - \alpha\frac{\mu_{(1)}^{N_1}+\mu_{(1)}^{N_2}}{2}. \tag{19}$$

Because $\widetilde{\xi}_{(1)}^{N_1+N_2}$ first-order stochastically dominates $\widetilde{\xi}_{(1)}^{N_m}$ for $m \in \{1,2\}$ (and not vice versa), we have $\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_m} > 0$ for $m \in \{1,2\}$, so $\overline{\Pi} - \overline{\Pi}^X \geq 0$ if $\alpha \geq \alpha_0 \equiv \max\{\alpha_1, \alpha_2\}$, where

$$\alpha_2 \equiv \frac{\theta}{bp}\frac{\log\left(I_{N_1}I_{N_2}\right) - 2\log\left(2^{1-b}I_{N_1+N_2}\right)}{2\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_1} - \mu_{(1)}^{N_2}}.$$

To show the second part of the result, suppose that $\overline{E}$ is sufficiently large so that $\alpha_1 < \alpha_2$. From the above discussion, we have $\overline{\Pi} - \overline{\Pi}^X \geq 0$ if $\alpha \geq \max\{\alpha_1, \alpha_2\} = \alpha_2$. Furthermore, from (19), we see that if $\alpha < \alpha_2$, $\overline{\Pi} - \overline{\Pi}^X < 0$. Therefore, $\overline{\Pi} - \overline{\Pi}^X \geq 0$ if and only if $\alpha \geq \alpha_0 = \alpha_2$. ■

Proof of Theorem 2. Let $\Phi(A)$ be defined as in Lemma 2. Note that

$$\Phi(\overline{A}) = r'\left(g\left(\overline{A}I_N M^{1-b}\right)\right)g'\left(\overline{A}I_N M^{1-b}\right)I_N M^{1-b} - 1 = r'\left(\overline{E}/M\right)g'\left(g^{-1}(\overline{E}/M)\right)I_N M^{1-b} - 1 \tag{20}$$

is increasing in M because $r'(g(x))g'(x)$ is decreasing in x and M^{1-b} is increasing in M. Because $\Phi(\overline{A})$ is increasing in M and $\lim_{x\to 0} r'(g(x))g'(x) = \infty$, there exists $M_0 \in [1,\infty)$ such that $\Phi(\overline{A}) < 0$ for any $M < M_0$, and $\Phi(\overline{A}) \geq 0$ for any $M \geq M_0$.

We next show that the average profit $\overline{\Pi} = \Pi^* = r(e^*) + \mu_{(1)}^N - A^*$ is increasing in the number of contests M up to some M^* and decreasing afterwards. When $M < M_0$, from Lemma 2 and the above discussion, the constraint in (15) can be relaxed. Applying the Envelope Theorem to $\overline{\Pi} \equiv \max_A r(e^*) + \mu_{(1)}^N - A$, we obtain

$$\frac{\partial\overline{\Pi}}{\partial M} = r'(e^*)\frac{\partial e^*}{\partial M} = (1-b)r'(e^*)g'\left(A^*I_N M^{1-b}\right)A^*I_N M^{-b}. \tag{21}$$

Because g is increasing, $g' > 0$, and because r is increasing, $r' > 0$. Thus, from (21), $\overline{\Pi}$ is increasing in M when $M < M_0$. When $M \geq M_0$, $A^* = \overline{A}$, so the average profit becomes

$$\overline{\Pi} = r\left(\frac{\overline{E}}{M}\right) + \mu_{(1)}^N - \frac{1}{I_N M^{1-b}}g^{-1}\left(\frac{\overline{E}}{M}\right). \tag{22}$$

The derivative of the average profit $\overline{\Pi}$ with respect to M

$$\frac{\partial\overline{\Pi}}{\partial M} = -r'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{1}{I_N M^{1-b}}(g^{-1})'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{1-b}{I_N M^{2-b}}g^{-1}\left(\frac{\overline{E}}{M}\right). \tag{23}$$

As r', η, and ϕ are homogeneous of degree $-k$, b, and p, respectively, $g^{-1} = \left(\frac{(\eta\circ\phi)'}{r'}\right)$ is homogeneous of degree $pb+k-1$. Noting that $(g^{-1})'(x) = (pb+k-1)g^{-1}(x)/x$, we can write (23) as

$$\frac{\partial\overline{\Pi}}{\partial M} = -r'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{pb+k-b}{I_N M^{2-b}} g^{-1}\left(\frac{\overline{E}}{M}\right) = r'\left(\frac{\overline{E}}{M}\right)\frac{1}{M^2}\left(-\overline{E} + \frac{pb+k-b}{I_N M^{-b}}\frac{g^{-1}}{r'}\left(\frac{\overline{E}}{M}\right)\right).$$

Note that $\frac{\partial\overline{\Pi}}{\partial M}$ has the same sign as

$$\varsigma \equiv -\overline{E} + \frac{pb+k-b}{I_N M^{pb+2k-1-b}}\frac{g^{-1}}{r'}\left(\overline{E}\right), \tag{24}$$

which is always decreasing in M because $pb+k-b>0$ and $pb+2k-1-b \geq 2-1-b > 0$ (note that $pb+2k-2\geq 0$). Thus, there exists $M^* \in [M_0,\infty)$ such that $\varsigma > 0$ and hence $\frac{\partial\overline{\Pi}}{\partial M} > 0$ for all $M \in [M_0, M^*)$; and $\varsigma < 0$ and hence $\frac{\partial\overline{\Pi}}{\partial M} < 0$ for all $M > M^*$. Because we also established above that $\frac{\partial\overline{\Pi}}{\partial M} > 0$ for all $M < M_0$, we have $\frac{\partial\overline{\Pi}}{\partial M} > 0$ for all $M < M^*$ and $\frac{\partial\overline{\Pi}}{\partial M} < 0$ for all $M > M^*$. ■

Proof of Corollary 2. Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a parameter $\alpha > 0$. After the transformation, $\widehat{I_N} = I_N/\alpha$. Thus, for any M, $\Phi(\overline{A})$ in (20) is decreasing in α, so M_0 in the proof of Theorem 2 is non-decreasing in α (increasing in α if $M_0 > 1$). Because ς in (24) is also increasing in α, $M^*(>1)$ is increasing in α. ■

Proof of Proposition 1. Because r', η, and ϕ are homogeneous of degree $-k$, b, and p, respectively, $g = ((\eta\circ\phi)'/r')^{-1}$ is homogeneous of degree $1/(bp+k-1)$. Thus, we can rewrite an agent's utility when she participates in M contests as (note that because we assume $\overline{E}$ is sufficiently large, the equilibrium effort $e^* = g(AI_N M^{1-b})$)

$$\begin{aligned} U[M] &= \frac{AM}{N} - \eta(M\phi(e^*)) - Mc_f = \frac{AM}{N} - \eta(M\phi(g(M^{1-b}AI_N))) - Mc_f \\ &= \frac{AM}{N} - M^{b+\frac{(1-b)bp}{bp+k-1}}\eta(\phi(g(AI_N))) - Mc_f = \frac{AM}{N} - M^{\frac{bp+b(k-1)}{bp+k-1}}\eta(\phi(g(AI_N))) - Mc_f. \end{aligned}$$

The derivative of $U[M]$ with respect to M

$$\begin{aligned} \frac{\partial U[M]}{\partial M} &= \frac{A}{N} - \frac{bp+b(k-1)}{bp+k-1}M^{\frac{(b-1)(k-1)}{bp+k-1}}\eta(\phi(g(AI_N))) - c_f \\ &= U[1] - \left(\frac{bp+b(k-1)}{bp+k-1}M^{\frac{(1-b)(1-k)}{bp+k-1}} - 1\right)\eta(\phi(g(AI_N))). \end{aligned}$$

(a) Because $U[1] > 0$, when $k \geq 1$, we have $\frac{bp+b(k-1)}{bp+k-1} < 1$ and $\frac{(1-b)(1-k)}{bp+k-1} < 0$. Because $M \geq 1$, we have $\partial U[M]/\partial M > 0$. Thus, $U[M] > 0$ for all M.

(b) Suppose that $k < 1$. Then, $\frac{bp+b(k-1)}{bp+k-1} > 1$, and in turn, $\lim_{M\to\infty} U[M]/M = -\infty$ and $U[M]/M$ is decreasing in M. Thus, there exists a unique $\overline{M}$ such that $U[\overline{M}] = 0$, and $U[M] < 0$ for all $M > \overline{M}$. Furthermore, when the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a parameter $\alpha > 0$, the agent's utility becomes $U[M] = \frac{AM}{N} - M^{\frac{bp+b(k-1)}{bp+k-1}}\eta(\phi(g(AI_N/\alpha))) - Mc_f$. Thus, the agent's utility is increasing in α, which means that $\overline{M}$ is increasing in α. ■

Proof of Proposition 2. Consider two contests with N_1 and N_2 agents and suppose that $\overline{E}$ is sufficiently large. Each organizer's profit is

$$\Pi_1^{*,L} = \frac{\theta}{bp}\log\left(\frac{\theta^2 I_{N_1}}{cb^2p^2}\right) + \mu_{(1)}^{N_1} - \frac{\theta}{bp} \text{ and } \Pi_2^{*,L} = \frac{\theta}{bp}\log\left(\frac{\theta^2 I_{N_2}}{cb^2p^2}\right) + \mu_{(1)}^{N_2} - \frac{\theta}{bp}.$$

The average profit under two contests is

$$\overline{\Pi}^{L,II} = \frac{1}{2}\left[\frac{\theta}{bp}\log\left(\frac{\theta^4 I_{N_1} I_{N_2}}{c^2b^4p^4}\right) + \mu_{(1)}^{N_1} + \mu_{(1)}^{N_2} - \frac{2\theta}{bp}\right].$$

The average profit under a single contest with N_1+N_2 agents is

$$\overline{\Pi}^{L,I} = \frac{\theta}{bp}\log\left(\frac{\theta^2 I_{N_1+N_2}}{cb^2p^2}\right) + \mu_{(1)}^{N_1+N_2} - \frac{\theta}{bp}.$$

When the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a parameter $\alpha > 0$, the difference between the average profit under two contests and that under a single contest is

$$\overline{\Pi}^{L,II} - \overline{\Pi}^{L,I} = \frac{\theta}{2bp}\log\left(\frac{I_{N_1} I_{N_2}}{I_{N_1+N_2}^2}\right) + \alpha\frac{\mu_{(1)}^{N_1} + \mu_{(1)}^{N_2}}{2} - \alpha\mu_{(1)}^{N_1+N_2}.$$

Noting that $\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_m} > 0$ for $m \in \{1,2\}$ because $\widetilde{\xi}_{(1)m}^{N_1+N_2}$ first-order stochastically dominates $\widetilde{\xi}_{(1)m}^{N_m}$ (and not vice versa), $\overline{\Pi}^{L,II} - \overline{\Pi}^{L,I} > 0$ if and only if $\alpha < \alpha_L$, where

$$\alpha_L \equiv \frac{\theta}{bp}\frac{\log\left(I_{N_1} I_{N_2}\right) - 2\log\left(I_{N_1+N_2}\right)}{2\mu_{(1)}^{N_1+N_2} - \mu_{(1)}^{N_1} - \mu_{(1)}^{N_2}}. \blacksquare$$

Proof of Corollary 3. Because the number of contests is fixed in Theorem 1, whenever the average profit is maximized, the total profit is also maximized. Thus, Theorem 1 directly extends to the case where the coordinator maximizes the total profit. ■

Proof of Proposition 3. As it is optimal for the coordinator to set equal awards at all contests by Lemma EC.5 of Online Appendix, without loss of optimality, the coordinator's problem is

$$\max_A \; Mr(e^*) + M\mu_{(1)}^N - MA, \text{ where } e^* = \min\{g(AI_N M^{1-b}), \overline{E}/M\}. \tag{25}$$

From the above problem, we can deduce that the coordinator never sets A such that $g(AI_NM^{1-b}) > \overline{E}/M$ because otherwise the coordinator can improve the total profit by reducing A. Thus, without loss of optimality, the coordinator's problem can be rewritten as follows:

$$\max_A \; Mr(g(AI_NM^{1-b})) + M\mu_{(1)}^N - MA, \text{ where } Mg(AI_NM^{1-b}) \le \overline{E}. \tag{26}$$

Let $\Phi(A) = Mr'\left(g\left(AI_NM^{1-b}\right)\right)g'\left(AI_NM^{1-b}\right)I_NM^{1-b} - M$ and $\overline{A} = M^{b-1}g^{-1}\left(\overline{E}/M\right)/I_N$. Note that Φ is the first derivative of (25) with respect to A. Suppose that $\Phi(\overline{A}) \ge 0$. Because $r'(g(x))g'(x)$ is decreasing in x (as assumed in §2), the objective function in (26) is concave in A, and hence $\Phi(A)$ is decreasing in A. Because $A > \overline{A}$ violates the constraint in (26), and Φ is decreasing, $A^* = \overline{A}$ solves (26). Thus, $A_m = A^*$ maximizes the total profit $\Pi^{*,\Sigma}$, and $e_m^* = e^* = \overline{E}/M$ is the corresponding equilibrium effort. Suppose that $\Phi(\overline{A}) < 0$. Then, as $\lim_{x\to 0} r'(g(x))g'(x) = \infty$, we have $\Phi(0) > 0$, so by the Intermediate Value Theorem, there exists $\widehat{A}$ such that $\Phi(\widehat{A}) = 0$. Note that $\widehat{A}$ is unique because Φ is decreasing. In this case, $A^* = \widehat{A}$ solves (26). Thus, $A_m = A^* = \widehat{A}$ maximizes the total profit $\Pi^{*,\Sigma}$, and $e_m^* = e^* = g(A^* I_N M^{1-b})$ is the corresponding equilibrium effort.

$\Phi(\overline{A})/M = r'\left(g\left(\overline{A} I_N M^{1-b}\right)\right) g'\left(\overline{A} I_N M^{1-b}\right) I_N M^{1-b} - 1 = r'\left(\overline{E}/M\right) g'\left(g^{-1}(\overline{E}/M)\right) I_N M^{1-b} - 1$ is increasing in M because $r'(g(x))g'(x)$ is decreasing in x and M^{1-b} is increasing in M. Because $\Phi(\overline{A})/M$ is increasing in M and $\lim_{x\to 0} r'(g(x))g'(x) = \infty$ and hence $\lim_{M\to\infty}(\Phi(\overline{A})/M) > 0$, there exists $M_0 \in [1, \infty)$ such that for any $M < M_0$, $\Phi(\overline{A}) < 0$ and for any $M \geq M_0$, $\Phi(\overline{A}) \geq 0$.

Let $\Pi^{*,M}$ be an organizer's profit when there are M contests and the coordinator optimally chooses the award as A^*. We next show that the total profit $\Pi^{*,\Sigma} = M\Pi^{*,M} = Mr(e^*) + M\mu_{(1)}^N - MA^*$ is increasing in the number of contests M up to some M^* and decreasing afterwards. When $M < M_0$ as in the proof of Theorem 2, the constraint in (26) can be relaxed. Applying the Envelope Theorem to $\Pi^{*,\Sigma} \equiv \max_A \; Mr(e^*) + M\mu_{(1)}^N - MA$, we obtain

$$\frac{\partial \Pi^{*,\Sigma}}{\partial M} = \Pi^{*,M} + Mr'(e^*)\frac{\partial e^*}{\partial M} = \Pi^{*,M} + M(1-b)r'(e^*)g'\left(A^* I_N M^{1-b}\right) A^* I_N M^{-b}. \tag{27}$$

A^* that maximizes $\Pi^{*,\Sigma}$ also maximizes $\Pi^{*,M}$, and we have $\frac{\partial \Pi^{*,M}}{\partial M} > 0$ for all $M < M_0$ (see proof of Theorem 2), so under Assumption 2, we have $\Pi^{*,M} > \Pi^{*,1} > 0$. As $g' > 0$ and $r' > 0$, from (27), $\Pi^{*,\Sigma}$ is increasing in M when $M < M_0$. When $M \geq M_0$, $A^* = \overline{A}$ so the objective function in (26) is

$$\Pi^{*,\Sigma} = \; Mr\left(\frac{\overline{E}}{M}\right) + M\mu_{(1)}^N - \frac{M}{I_N M^{1-b}} g^{-1}\left(\frac{\overline{E}}{M}\right). \tag{28}$$

The derivative of the coordinator's objective with respect to M

$$\frac{\partial \Pi^{*,\Sigma}}{\partial M} = \; \Pi^{*,M} + M\frac{\partial \Pi^{*,M}}{\partial M} = \; \Pi^{*,M} + Mr'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{M}{I_N M^{1-b}}(g^{-1})'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{M(1-b)}{I_N M^{2-b}} g^{-1}\left(\frac{\overline{E}}{M}\right).$$

As r', ϕ, and η are homogeneous of degree $-k$, p, and b, respectively, $g^{-1} = \left(\frac{(\eta\circ\phi)'}{r'}\right)$ is homogeneous of degree $pb + k - 1$. Noting that $(g^{-1})'(x) = (pb + k - 1)g^{-1}(x)/x$, we have

$$\frac{\partial \Pi^{*,M}}{\partial M} = \; -r'\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2} + \frac{pb+k-b}{I_N M^{2-b}} g^{-1}\left(\frac{\overline{E}}{M}\right) = r'\left(\frac{\overline{E}}{M}\right)\frac{1}{M^2}\left(-\overline{E} + \frac{pb+k-b}{I_N M^{-b}}\frac{g^{-1}}{r'}\left(\frac{\overline{E}}{M}\right)\right).$$

Note that $\frac{\partial \Pi^{*,M}}{\partial M}$ has the same sign as $\varsigma \equiv -\overline{E} + \frac{pb+k-b}{I_N M^{pb+2k-1-b}}\frac{g^{-1}}{r'}\left(\overline{E}\right)$, which is always decreasing in M because $(p-1)b + k > 0$ and $pb + 2k - 1 - b \geq 2 - 1 - b > 0$ (note that $pb + 2k \geq 2$). Thus, there exists $M_1 \in [M_0, \infty)$ such that $\varsigma > 0$ and hence $\frac{\partial \Pi^{*,M}}{\partial M} > 0$ for all $M \in [M_0, M_1)$; and $\varsigma < 0$ and hence $\frac{\partial \Pi^{*,M}}{\partial M} < 0$ for all $M > M_1$. Then, as $\Pi^{*,M} > 0$ for all $M < M_0$, and $\frac{\partial \Pi^{*,M}}{\partial M} > 0$ for all $M \in [M_0, M_1)$, we have $\Pi^{*,M} > 0$ for all $M < M_1$. For $M > M_1$, $\frac{\partial \Pi^{*,M}}{\partial M} < 0$, and hence $\Pi^{*,M}$ is decreasing in M. Thus, there exists $M^{*,\Sigma}$ such that $\frac{\partial \Pi^{*,\Sigma}}{\partial M} > 0$ for all $M < M^{*,\Sigma}$ and $\frac{\partial \Pi^{*,\Sigma}}{\partial M} < 0$ for all $M > M^{*,\Sigma}$. Also, because $r(e^*) = r(1) + \int_1^{e^*} r'(e)de = r(1) + \int_1^{e^*} e^{-k} r'(1)de = r(1) + r'(1)\frac{(e^*)^{1-k} - 1}{1-k}$, for $k \geq 1$, we have $\lim_{M\to\infty} \Pi^{*,M} = \lim_{M\to\infty}\left(r(1) + r'(1)\frac{(\overline{E}/M)^{1-k} - 1}{1-k} + \mu_{(1)}^N - \frac{1}{I_N M^{1-b}} g^{-1}\left(\frac{\overline{E}}{M}\right)\right) = -\infty$, so $M^{*,\Sigma} \in \mathbb{R}_+$. ■

Proof of Proposition 4. We find the symmetric equilibrium in the decentralized case, and then prove parts (a) and (b), respectively.

We first find the unconstrained decentralized award by relaxing the agent's capacity constraint, which we denote by $\widehat{A}$. Suppose that each organizer $k \neq m$ chooses $\widehat{A}$ and organizer m chooses A.

Let e be the agent's effort at contest m and let $\widehat{e}$ be the unconstrained equilibrium effort at other contests. In this case, using (4), the first-order conditions for the agent can be written as

$$\widehat{A}r'(\widehat{e})I_N - \eta'\left((M-1)\phi(\widehat{e}) + \phi(e)\right)\phi'(\widehat{e}) = 0,$$
$$Ar'(e)\,I_N - \eta'\left((M-1)\phi(\widehat{e}) + \phi(e)\right)\phi'(e) = 0.$$

Under Assumption 1, from the above equalities, we can derive the following equalities:

$$A\frac{\theta}{e^p}I_N = \widehat{A}\frac{\theta}{(\widehat{e})^p}I_N = cbp\left((M-1)(\widehat{e})^p + e^p\right)^{b-1}.$$

From the first equality, we get $e^p = \frac{A}{\widehat{A}}(\widehat{e})^p$, and by plugging this into the second equality, we get

$$\widehat{A}\frac{\theta}{(\widehat{e})^p}I_N = b\left((M-1)(\widehat{e})^p + \frac{A}{\widehat{A}}(\widehat{e})^p\right)^{b-1} = cbp(\widehat{e})^{(b-1)p}\left(\frac{(M-1)\widehat{A}+A}{\widehat{A}}\right)^{b-1},$$

which yields $\widehat{e} = (\widehat{A})^{\frac{1}{p}}\left(\frac{\theta I_N}{cbp\left((M-1)\widehat{A}+A\right)^{b-1}}\right)^{\frac{1}{pb}}$ and $e = A^{\frac{1}{p}}\left(\frac{\theta I_N}{cbp\left((M-1)\widehat{A}+A\right)^{b-1}}\right)^{\frac{1}{pb}}$.

While other organizers choose $\widehat{A}$, organizer m's profit (when he chooses A) can be written as

$$\Pi_m(A,\widehat{A}) = \frac{\theta}{p}\log(A) + \frac{\theta}{bp}\log\left(\frac{\theta I_N}{cbp}\right) + \frac{\theta(1-b)}{bp}\log\left((M-1)\widehat{A} + A\right) + \mu_{(1)}^N - A.$$

A necessary condition for $\widehat{A}$ to be unconstrained equilibrium is

$$\left.\frac{\partial \Pi_m(A,\widehat{A})}{\partial A}\right|_{A=\widehat{A}} = \frac{\theta}{p}\left.\left(\frac{1}{\widehat{A}} + \frac{1-b}{b}\frac{1}{A+(M-1)\widehat{A}}\right)\right|_{A=\widehat{A}} - 1 = \frac{\theta}{p}\left(\frac{1}{\widehat{A}} + \frac{1-b}{b}\frac{1}{M\widehat{A}}\right) - 1 = 0,$$

which yields

$$\widehat{A} = \frac{\theta(1-b+Mb)}{Mbp} = \frac{\theta\left(M + \frac{1-b}{b}\right)}{Mp}.$$

Let $\overline{A} = M^{b-1}g^{-1}\left(\overline{E}/M\right)/I_N = M^{b(1-p)-1}\frac{cbp}{\theta I_N}\left(\overline{E}\right)^{bp}$. Note that if all organizers give award $\overline{A}$, then the agent's equilibrium effort at each contest is $\overline{E}/M$. Suppose that $\widehat{A} < \overline{A}$. The agent's capacity constraint is satisfied by the unconstrained equilibrium, and hence the equilibrium award in the decentralized case is $A^{*,D} = \widehat{A}$. Note that all organizers' giving awards $\overline{A}$ is not an equilibrium as

$$\frac{\theta}{p}\left(\frac{1}{\overline{A}} + \frac{1-b}{b}\frac{1}{M\overline{A}}\right) - 1 < \frac{\theta}{p}\left(\frac{1}{\widehat{A}} + \frac{1-b}{b}\frac{1}{M\widehat{A}}\right) - 1 = 0,$$

which shows that an organizer improves his profit by reducing his award.

Suppose that $\widehat{A} \geq \overline{A}$. In this case, any award $A < \overline{A}$ cannot be an equilibrium award because

$$\frac{\theta}{p}\left(\frac{1}{A} + \frac{1-b}{b}\frac{1}{MA}\right) - 1 > \frac{\theta}{p}\left(\frac{1}{\overline{A}} + \frac{1-b}{b}\frac{1}{M\overline{A}}\right) - 1 \geq \frac{\theta}{p}\left(\frac{1}{\widehat{A}} + \frac{1-b}{b}\frac{1}{M\widehat{A}}\right) - 1 = 0,$$

which indicates that an organizer has an incentive to increase the award above A. Suppose all other organizers give award $\check{A}$, where $\check{A} \geq \overline{A}$, and let $\check{e}$ be the corresponding equilibrium effort. In this case, when an organizer selects award A such that the agent's capacity constraint binds, we have

$$\check{A}r'(\check{e})I_N - \eta'\left((M-1)\phi(\check{e}) + \phi(e)\right)\phi'(\check{e}) = Ar'(e)\,I_N - \eta'\left((M-1)\phi(\check{e}) + \phi(e)\right)\phi'(e). \tag{29}$$

Under Assumption 1, the above relationship becomes

$$\check{A}\frac{\theta}{\check{e}}I_N - cbp\left((M-1)(\check{e})^p + e^p\right)^{b-1}(\check{e})^{p-1} = A\frac{\theta}{e}I_N - cbp\left((M-1)(\check{e})^p + e^p\right)^{b-1}(e)^{p-1}.$$

Using the relationship $e+(M-1)\check{e}=\overline{E}$ yields

$$\check{A}\frac{\theta(M-1)}{\overline{E}-e}I_N - A\frac{\theta}{e}I_N - cbp((M-1)^{1-p}(\overline{E}-e)^p+e^p)^{b-1}\left[\left(\frac{\overline{E}-e}{M-1}\right)^{p-1}-e^{p-1}\right]=0. \tag{30}$$

Applying the Implicit Function Theorem on (30) evaluated at $A=\check{A}$ and $e=\overline{E}/M$ yields

$$\left.\frac{de}{dA}\right|_{A=\check{A}} = \frac{\frac{\theta M}{\overline{E}}I_N}{\frac{M}{M-1}\check{A}\frac{\theta M^2}{(\overline{E})^2}I_N + \frac{M}{M-1}(p-1)cbp((M^{1-p}(\overline{E})^p)^{b-1}(\overline{E})^{p-2}M^{2-p}}. \tag{31}$$

Organizer m's first-order condition evaluated at $A=\check{A}$ can be written as

$$\left.\frac{\partial\Pi_m(A,\check{A})}{\partial A}\right|_{A=\check{A}} = \frac{\theta M}{\overline{E}}\left.\frac{de}{dA}\right|_{A=\check{A}} - 1 = 0, \tag{32}$$

which yields the equilibrium award in the decentralized case $A^{*,D}$ as

$$A^{*,D}=\check{A}=\frac{M-1}{M}\theta - \frac{(p-1)cbp}{\theta I_N}M^{b-pb-1}(\overline{E})^{bp}. \tag{33}$$

(**a**) We next compare the average profit in the non-exclusive decentralized case with that in the exclusive case. Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ with $\alpha>0$. Note that $\overline{A}$ increases with the parameter α as $\widehat{I}_N=I_N/\alpha$ increases with α. As $\widehat{A}$ does not depend on α, there exists $\overline{\alpha}$ such that for all $\alpha>\overline{\alpha}$, the equilibrium award in the decentralized case is $A^{*,D}=\widehat{A}=\frac{\theta\left(M+\frac{1-b}{b}\right)}{Mp}$. The average profit in the non-exclusive decentralized case is

$$\overline{\Pi}^D=\frac{\theta}{bp}\log\left(\frac{\theta^2 I_N M^{1-b}(Mb-b+1)}{\alpha cb^2p^2M}\right)+\alpha\mu^N_{(1)}-\frac{\theta(Mb-b+1)}{Mbp}. \tag{34}$$

The equilibrium effort in the exclusive case is $e^{*,X}_m=\left(\frac{\theta A^{*,X}_m I_{N^{*,X}_m}}{cbp}\right)^{\frac{1}{bp}}$, where the optimal award $A^{*,X}_m=\frac{\theta}{bp}$, for $m\in\{1,2\}$. Then, the average profit in the exclusive case is

$$\overline{\Pi}^X=\frac{1}{2}\sum_{m=1}^{2}\left(\frac{\theta}{bp}\log\left(\frac{\theta^2 I_{N^{*,X}_m}}{\alpha cb^2p^2}\right)+\alpha\mu^{N^{*,X}_m}_{(1)}-\frac{\theta}{bp}\right). \tag{35}$$

The difference between the average profit in non-exclusive decentralized and exclusive cases is

$$\overline{\Pi}^D-\overline{\Pi}^X=\frac{\theta}{bp}\log\left(\frac{I_N\,(2b-b+1)}{2^b(I_{N^{*,X}_1}I_{N^{*,X}_2})^{1/2}}\right)+\alpha\left(\mu^N_{(1)}-\frac{1}{2}\sum_{m=1}^{2}\mu^{N^{*,X}_m}_{(1)}\right)-\frac{\theta(b-1)}{2b}. \tag{36}$$

As $N>N^{*,X}_m$ for a sufficiently large α (see Lemma EC.10 of Online Appendix), $\widetilde{\xi}^N_{(1)_m}$ first-order stochastically dominates $\widetilde{\xi}^{N_m}_{(1)m}$ for $m\in\{1,2\}$ (and not vice versa), so we have $\mu^N_{(1)}>\frac{1}{2}\sum_{m=1}^{2}\mu^{N^{*,X}_m}_{(1)}$ for $m\in\{1,2\}$. Thus, for a sufficiently large α, $\overline{\Pi}^D-\overline{\Pi}^X>0$, so there exists α_0 $(\geq\overline{\alpha})$ such that for any scale transformation $\widehat{\xi}_{im}=\alpha\widetilde{\xi}_{im}$ of the output shock $\widetilde{\xi}_{im}$ with $\alpha>\alpha_0$, $\overline{\Pi}^D$ is greater than $\overline{\Pi}^X$.

(**b**) From part (**a**), for $\overline{A}=M^{b(1-p)-1}\frac{cbp}{\theta I_N}\left(\overline{E}\right)^{bp}$, we know that the equilibrium award in the decentralized case is $A^{*,D}=\widehat{A}$ if $\widehat{A}<\overline{A}$ and $A^{*,D}=\check{A}$ if $\check{A}>\overline{A}$. First, we analyze when $\widehat{A}<\overline{A}$ holds and then we analyze when $\check{A}>\overline{A}$ holds. By rearranging, we obtain $\widehat{A}<\overline{A}$ if and only if

$$\theta^2MI_Nb+\theta^2(1-b)I_N<M^{b-bp}cb^2p^2\left(\overline{E}\right)^{bp}.$$

Since $b<1$ and $bp\geq1$, we have $b-bp\leq0$. Then, the left-hand side is increasing in M, and the right-hand side is decreasing in M, and hence when M is sufficiently small (possibly <1), the equilibrium award $A^{*,D} = \widehat{A}$. Thus, there exists a threshold $M_1^{*,D}$ such that for all $M < M_1^{*,D}$, we have $A^{*,D} = \widehat{A}$. To have $M_1^{*,D} > 1$, we need $\theta < \left(\frac{cb^2p^2\left(\overline{E}\right)^{bp}}{1+I_N}\right)^{\frac{1}{2}}$.

Similarly to above, by rearranging, we obtain $\check{A} > \overline{A}$ if and only if $M\theta - \theta > M^{b-bp}\frac{cbp^2}{\theta I_N}\left(\overline{E}\right)^{bp}$. Since $bp \geq 1$, the left-hand side is increasing in M and the right-hand side is decreasing in M, and hence when M is sufficiently large, the equilibrium award $A^{*,D} = \check{A}$. Therefore, there exists $M_2^{*,D}$ such that for all $M > M_2^{*,D}$, we have $A^{*,D} = \check{A}$.

We next show that when $M < M_1^{*,D}$, so $A^{*,D} = \widehat{A}$, we have $\frac{\partial \overline{\Pi}^D}{\partial M} > 0$. Note that we have

$$\overline{\Pi}^D = \frac{\theta}{bp}\log\left(\frac{\theta^2 I_N M^{1-b}(Mb-b+1)}{\alpha cb^2p^2M}\right) + \mu_{(1)}^N - \frac{\theta(Mb-b+1)}{Mbp}.$$

As $b < 1$, $-\frac{\theta(Mb-b+1)}{Mbp}$ is increasing in M. Also, $\frac{M^{1-b}(Mb-b+1)}{M}$ is increasing in M for $M \geq 1$. Thus, we have $\frac{\partial \overline{\Pi}^D}{\partial M} > 0$ for all $M < M_1^{*,D}$. We then show that when $M > M_2^{*,D}$, we have $A^{*,D} = \check{A}$, and hence $\frac{\partial \overline{\Pi}^D}{\partial M} < 0$. Note that we have $e^* = \check{e} = \overline{E}/M$. Thus, $\overline{\Pi}^D = \frac{\theta}{bp}\log\left(\frac{\overline{E}}{M}\right) + \mu_{(1)}^N - \check{A}$, which is decreasing in M, because $\check{A}$ in (33) is increasing in M. ■

Proof of Proposition 5. We first characterize the agent's equilibrium effort, and then prove the two parts of the proposition. Agent i solves the following problem:

$$\max_{e_{i1},e_{i2},\ldots,e_{iM}} \sum_{m=1}^{M} A_m \int H\left(s + (1-a)e_{im} + \sum_{l=1}^{M} ae_{il} - (1-a)e_m^* - \sum_{l=1}^{M} ae_l^*\right)^{N-1} h(s)ds - \sum_{m=1}^{M}\phi(e_{im}),$$
$$\text{s.t.} \quad \sum_{m=1}^{M} e_{im} \leq \overline{E}.$$

When the agent's capacity constraint is relaxed, the first-order conditions of the above problem evaluated at symmetric equilibrium yields $\widehat{e}_m = (\phi')^{-1}\left(\left((1-a)A_m + a\sum_{l=1}^{M} A_l\right)I_N\right)$. When all contests give award A, the agent's equilibrium effort considering her capacity constraint is

$$e_m^* = \min\left\{(\phi')^{-1}\left(A\left(1 + a(M-1)\right)I_N\right), \frac{\overline{E}}{M}\right\}. \tag{37}$$

(a) We prove the first part of the result for two contests but the result can be generalized to any number of contests $M > 2$. We compare the average profit in the exclusive and non-exclusive cases. In the exclusive case, let $N_m^{*,X}$ be the optimal number of agents and $A_m^{*,X}$ be the optimal award at contest $m \in \{1,2\}$, and let $e_m^{*,X}$ be the corresponding equilibrium effort at contest $m \in \{1,2\}$. Note that it is never optimal for the coordinator to set awards such that $e_m^{*,X} > \overline{E}$ (for $m \in \{1,2\}$) because the coordinator can improve the average profit by reducing the award at contest $m \in \{1,2\}$. Thus, the equilibrium effort in the exclusive case is $e_m^{*,X} = (\phi')^{-1}(A_m^{*,X} I_{N_m^{*,X}})$ at contest $m \in \{1,2\}$. Without loss of generality, suppose that $e_1^{*,X} \leq e_2^{*,X}$. After incorporating the optimal solution, the average profit in the exclusive case becomes

$$\overline{\Pi}^X = \frac{1}{2}\sum_{m=1}^{2}(\theta e_m^{*,X} + \mu_{(1)}^{N_m^{*,X}} - A_m^{*,X}). \tag{38}$$

In the non-exclusive case, suppose that the coordinator offers an award A at each contest so that the equilibrium effort at each contest $m \in \{1,2\}$ is $e_m^* = (e_1^{*,X} + e_2^{*,X})/2$. Note that because $\sum_{m=1}^2 e_m^* = (e_1^{*,X} + e_2^{*,X}) \le \overline{E}$, from (37), we know that A satisfies

$$A = \frac{1}{(1+a)\,I_N}\phi'\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right) \le \frac{1}{(1+a)\,I_N}\phi'(e_2^{*,X}) = \frac{A_2^{*,X} I_{N_2^{*,X}}}{(1+a)\,I_N}.$$

Using the above inequality, the average profit in the non-exclusive case becomes

$$\overline{\Pi} = (1+a)\,\theta\left(\frac{e_1^{*,X}+e_2^{*,X}}{2}\right) + \mu_{(1)}^N - A \ge \frac{1}{2}\sum_{m=1}^{2}(1+a)\,\theta e_m^{*,X} + \mu_{(1)}^N - \frac{A_2^{*,X} I_{N_2^{*,X}}}{(1+a)\,I_N}. \tag{39}$$

Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. The difference between the average profit in non-exclusive and exclusive cases satisfies

$$\overline{\Pi} - \overline{\Pi}^X \ge \alpha\left(\mu_{(1)}^N - \frac{1}{2}\sum_{m=1}^{2}\mu_{(1)}^{N_m^{*,X}} + \frac{1}{2}\sum_{m=1}^{2}\frac{A_m^{*,X}}{\alpha} - \frac{A_2^{*,X} I_{N_2^{*,X}}}{\alpha(1+a)I_N}\right). \tag{40}$$

As Lemma EC.7 of Online Appendix shows, $\lim_{\alpha\to\infty} A_m^{*,X}/\alpha = 0$ for each $m \in \{1,2\}$. Also, as $N > N_m^{*,X}$ for a sufficiently large α (as shown in Lemma EC.10 of Online Appendix), $\widetilde{\xi}_{(1)m}^N$ first-order stochastically dominates $\widetilde{\xi}_{(1)m}^{N_m}$ for $m \in \{1,2\}$ (and not vice versa), so we have $\mu_{(1)}^N > \frac{1}{2}\sum_{m=1}^2 \mu_{(1)}^{N_m^{*,X}}$ for $m \in \{1,2\}$. Thus, there exists α_0 such that for any $\alpha > \alpha_0$, we have $\overline{\Pi} - \overline{\Pi}^X > 0$.

(b) The average profit can be written as:

$$\begin{aligned}
\overline{\Pi} &= \frac{1}{M}\left[\sum_{m=1}^{M}\left((1-a)e_m^* + \sum_{l=1}^{M} a e_l^*\right) + \sum_{m=1}^{M} E[\widetilde{\xi}_{(1)m}^N] - \sum_{m=1}^{M} A_m\right] \\
&= \frac{1}{M}\left[\sum_{m=1}^{M}(1+(M-1)a)e_m^* + \sum_{m=1}^{M}\mu_{(1)}^N - \sum_{m=1}^{M} A_m\right] \\
&= \frac{1}{M}\left[\sum_{m=1}^{M}(1+(M-1)a)(\phi')^{-1}\left(\left((1-a)A_m + a\sum_{l=1}^{M} A_l\right) I_N\right) + \sum_{m=1}^{M}\mu_{(1)}^N - \sum_{m=1}^{M} A_m\right].
\end{aligned}$$

Due to the symmetry with respect to all contests and the concavity of $(\phi')^{-1}$ (which is guaranteed because $p > 2$), the coordinator sets the same award at each contest (otherwise the average profit can be improved by a perturbation that makes the awards equal with the same total award). Let $\overline{A} \equiv \frac{1}{I_N(1+(M-1)a)}\phi'\left(\frac{\overline{E}}{M}\right)$. Note from (37) that when the coordinator offers an award $\overline{A}$ at each contest, then the agent's total equilibrium effort is $\overline{E}$. The coordinator never chooses an award $A > \overline{A}$ because otherwise, the average profit can be improved by reducing awards marginally (and keeping the total effort as $\overline{E}$). Thus, the coordinator solves the following problem:

$$\overline{\Pi}(A^*) = \max_A \left[(1+(M-1)a)(\phi')^{-1}\left((1+(M-1)a)AI_N\right) + \mu_{(1)}^N - A\right] \text{ s.t. } A \le \overline{A}.$$

Let $\widehat{A}$ be the solution to the above problem when the constraint is relaxed. Note that because $(\phi')^{-1}$ is increasing, when ignoring the constraint, the Envelope Theorem implies that

$$\frac{\partial\overline{\Pi}(\widehat{A})}{\partial M} = a(\phi')^{-1}\left((1+(M-1)a)\widehat{A}I_N\right) + (1+(M-1)a)((\phi')^{-1})'\left((1+(M-1)a)\widehat{A}I_N\right) > 0.$$

Thus, the coordinator's objective improves with M if $\widehat{A} < \overline{A}$. Also, we can derive $\widehat{A}$ as

$$\widehat{A} = \frac{1}{(1+(M-1)a)I_N}(((\phi')^{-1})')^{-1}\left(\frac{1}{(1+(M-1)a)^2 I_N}\right) = (1+(M-1)a)^{\frac{p}{p-2}}(((\phi')^{-1})')^{-1}\left(\frac{1}{I_N}\right)\frac{1}{I_N},$$

which is increasing and unbounded in M because $p > 2$ and $\widehat{A} < \overline{A}$. Thus, there exists M_0 such that for all $M \geq M_0$, $\widehat{A} \geq \overline{A}$, and hence it is optimal for the coordinator to set $A^* = \overline{A}$. Therefore, for $M \geq M_0$, the coordinator's objective under the optimal award becomes

$$\overline{\Pi}(A^*) = (1+(M-1)a)\frac{\overline{E}}{M} + \mu_{(1)}^N - \frac{1}{I_N(1+(M-1)a)}\phi'\left(\frac{\overline{E}}{M}\right).$$

The derivative of the coordinator's objective with respect to M

$$\frac{\partial\overline{\Pi}(A^*)}{\partial M} = -(1-a)\frac{\overline{E}}{M^2} + \frac{a}{I_N(1+(M-1)a)^2}\phi'\left(\frac{\overline{E}}{M}\right) + \frac{1}{I_N(1+(M-1)a)}\phi''\left(\frac{\overline{E}}{M}\right)\frac{\overline{E}}{M^2}.$$

Note that $\frac{\partial\overline{\Pi}(A^*)}{\partial M}$ has the same sign as

$$\frac{M^2\partial\overline{\Pi}(A^*)}{\partial M} = -(1-a)\overline{E} + \frac{a\phi'(\overline{E})M}{I_N(1+(M-1)a)^2}M^{2-p} + \frac{\phi''(\overline{E})\overline{E}}{I_N(1+(M-1)a)}M^{2-p},$$

which is decreasing in M because $\frac{M}{(1+(M-1)a)^2}$ decreases with M and $p > 2$. Furthermore, $\lim_{M\to\infty}\frac{\partial\overline{\Pi}(A^*)M^2}{\partial M} = -(1-a)\overline{E}$, which means that there exists M^* such that for all $M > M^*$, we have $\frac{\partial\overline{\Pi}(A^*)}{\partial M} < 0$ and for all $M < M^*$ (where M^* can be equal to M_0), we have $\frac{\partial\overline{\Pi}(A^*)}{\partial M} > 0$. ■

Endnotes

1. Statistical analysis at InnoCentive reveals that in theoretical challenges where agents develop theoretical solutions with no implementation requirement, agents often work on multiple contests in parallel. Specifically, among four random days within the past twelve months, 57.4% of agents have opened more than one project room in live theoretical challenges in a day. Note that this number is likely to be a significant underestimation of the actual percentage of agents who have been working on multiple contests because this analysis does not account for agents who work on some contests offline and agents who allocate one day to one contest and the next day to another contest. We thank Graham Buchanan, marketing director at InnoCentive for sharing this statistic.

2. We thank John Elliott, business development manager at InnoCentive and Greg Bell, head of marketing and community at Topcoder for providing insights into their operations.

3. During our interviews with Topcoder, we have learned that development challenges that seek low-novelty solutions are designed to focus agents' efforts on a single contest. In algorithm challenges that seek innovative solutions, agents quite often work on multiple contests in parallel. We thank Clinton Bonner, director of marketing and crowdsourcing strategy at Topcoder for providing this information.

4. Several factors can contribute to economies of scope. For instance, Sutton (2001) mentions the following factors that lead to economies of scope in R&D: "There may be some common elements in the technologies employed along two different [research] trajectories, and know-how accumulated

along one trajectory may benefit the firm in its advance along some other trajectory" (page 24). For example, an agent at Topcoder can use the same programming language or the same code fragment at different contests she participates in.

5. For a detailed review of this literature and other types of contests, we refer the reader to Ales et al. (2017a). Our paper is broadly related to studies that consider heterogeneous agents by suppressing uncertainty (e.g., Moldovanu and Sela 2001, Körpeoğlu and Cho 2017, Stouras et al. 2017), to studies that analyze other types of contests (e.g., dynamic contests by Bimpikis et al. 2017), to empirical studies on crowdsourcing (e.g., Jiang et al. 2016), and to theoretical studies on new product development (e.g., Mihm 2010, Lobel et al. 2016), and to the price-competition literature (e.g., Federgruen and Hu 2016, Zhou et al. 2016).

6. Our paper is broadly related to the literature on multiple auctions. As a pioneering paper, McAfee (1993) show that in equilibrium, sellers hold identical auctions and buyers randomize over the sellers they visit. Peters and Severinov (1997) extend the McAfee (1993) model and analyze how reserve prices are determined. In the operations literature, Beil and Wein (2009) consider two competing auctioneers facing "pooled bidders" who can participate in both auctions as well as dedicated bidders. They show that for multi-item auctions, only the auctioneer with the smaller ratio of bidders per item benefits from the existence of pooled bidders. Not only do these papers address different research questions than ours, but also auctions have three fundamental differences than contests. First, while auction papers typically analyze settings with private information, contest papers analyze settings with moral hazard due to agents' unobservable efforts. Second, an auctioneer maximizes the total bid from bidders, whereas a contest organizer maximizes the quality of the best solution minus the total award. Finally, while the bids in an auction are deterministic, an agent's solution quality in a contest depends on an output uncertainty.

7. In practice, there may be some contest-specific dependence due to the uncertainty of the evaluation process. In this case, each agent i's output shock at contest m can be modeled as $\widetilde{\xi}_{im} + \widetilde{\epsilon}_m$, where $\widetilde{\epsilon}_m$ is a shock that is specific to contest m. Because $\widetilde{\epsilon}_m$ terms would appear in all agents' outputs, they would not affect agents' rankings or our analysis, and hence we omit them.

8. In practice, another plausible case is that such an organization determines whether to run contests in parallel or sequentially. As long as parallel contests create larger economies of scope than sequential contests, results under a sequential model would be qualitatively similar to our results. We provide a more detailed discussion of this case in §5.

9. In the exclusive case, we assume that the coordinator optimally determines awards and allocates agents to contests. Note that the average profit in this case is an upper bound for the average profit when each agent endogenously selects which contest to enter. Thus, our result directly applies to the case with endogenous entry as well.

10. Note that while Figure 1 illustrates a case where the capacity $\overline{E}$ on the agent's total effort is sufficiently large and where the award A^* does not change with the scale parameter α, Theorem 1 is not restricted to these cases.

11. In the innovation-contest literature, the agent's output uncertainty is often associated with the novelty of solutions (e.g., Terwiesch and Xu 2008). In particular, agents face small uncertainty in contests that seek low-novelty solutions, whereas they face large uncertainty in contests that seek innovative solutions. Nittala and Krishnan (2016) relate the agent's output uncertainty to how broadly an organizer defines a problem, which may be associated with how novel solutions an organizer seeks.

12. Alternatively, one may define the agent's participation condition as $U[M] = \max_{m\in\{1,2,\ldots,M\}}\{U[m]\}$. Proposition 1 holds under this definition as well.

13. We use the phrase "generally" because Ales et al. (2017b) show that the equilibrium effort e^* decreases with the number of agents N in a contest for most commonly used distributions for the output shock (e.g., exponential, Gumbel, logistic, or normal distribution).

14. It is plausible that the spillover from one contest may diminish with the spillover from other contests. This case can be modeled by taking the coefficient a as a decreasing function of M. Our results extend to this case as well.

15. It is likely that contests that are run in parallel feature larger economies of scope than sequential contests because when contests are run in parallel, the learning effect is two sided, whereas when contests are run sequentially, only the later contest benefits from the learning effect.

Online Appendix

EC.1. Existence of Equilibrium

The following three lemmas provide sufficient conditions for the concavity of the agent's utility function and show the existence and uniqueness of equilibrium for agents.

LEMMA EC.1. *Suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. For a sufficiently large α or $-r''/(r')^2$ or $\phi''/(\phi')^2$, there exists $\underline{b} < 1$ such that agent i's utility function $U_i = \sum_{m=1}^{M} A_m P_m(e_{im}, e_m^*) - \psi(e_{i1}, e_{i2}, ..., e_{iM})$ is concave in agent i's efforts $(e_{i1}, e_{i2}..., e_{iM})$ for all $b > \underline{b}$, where $P_m(e_{im}, e_m^*)$ is as in (2).*

Proof. For notational convenience, we drop e_m^* from $P_m(e_{im}, e_m^*)$. We derive a sufficient condition for U_i to be concave for $M = 2$, but the argument can be generalized. The Hessian matrix of U_i is

$$D^2U_i = \begin{bmatrix} B_1 - \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{i1}))^2 & \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi'(e_{i1})\phi'(e_{i2}) \\ \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi'(e_{i1})\phi'(e_{i2}) & B_2 - \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{i2}))^2 \end{bmatrix}, \tag{EC.1}$$

where $B_m = A_m P_m''(e_{im}) - \eta'\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi''(e_{im})$. U_i is concave if and only if D^2U_i is negative semi-definite, which is satisfied if $B_m - \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{im}))^2 \leq 0$ for $m \in \{1,2\}$ and $|D^2U_i| = B_1B_2 - B_2\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{i1}))^2 - B_1\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{i2}))^2 \geq 0$. The first condition is always satisfied because $\eta'\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi'(e_{im}) = \left(\frac{\phi(e_{im})}{\sum_{l=1}^{M}\phi(e_{il})}\right)^{1-b}\eta'(\phi(e_{im}))\phi'(e_{im})$ is increasing in e_{im}. A sufficient condition for $|D^2U_i| \geq 0$ is $B_m \leq 2\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{im}))^2$. Under a scale transformation of the output shock $\widetilde{\xi}_{im}$ to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$, $P_m(e_{im}) = \int_{s\in\Xi} H\left(s + \frac{r(e_{im}) - r(e_m^*)}{\alpha}\right)^{N-1} h(s)ds = E\left[H_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r(e_m^*)}{\alpha}\right)\right]$. The first derivative of $P_m(e_{im})$ is $P_m'(e_{im}) = \frac{r'(e_{im})}{\alpha}E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r(e_m^*)}{\alpha}\right)\right]$. Then, the second derivative of $P_m(e_{im})$ is (where $r_m^* = r(e_m^*)$)

$$P_m''(e_{im}) = \frac{r'(e_{im})^2}{\alpha^2}E\left[\left(h_{(1)}^{N-1}\right)'\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r_m^*}{\alpha}\right)\right] + \frac{r''(e_{im})}{\alpha}E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r_m^*}{\alpha}\right)\right]. \tag{EC.2}$$

There are three sufficient conditions for $B_m \leq 2\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{im}))^2$. When $B_m < 0$ for $m \in \{1,2\}$, there exists $\underline{b} < 1$ such that $B_m \leq 2\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{im}))^2$ for all m and $b > \underline{b}$. First, as α approaches infinity, both expectation terms in (EC.2) converge, and $E\left[h_{(1)}^{N-1}\left(\widetilde{\xi}_{im} + \frac{r(e_{im}) - r_m^*}{\alpha}\right)\right]$ converges to a positive constant. Furthermore, because r is increasing and concave, $r'(e_{im})^2/\alpha^2$ (> 0) approaches 0 faster than $r''(e_{im})/\alpha$ (< 0). Thus, for a sufficiently large α, $P_m''(e_{im}) < 0$. Second, $P_m''(e_{im}) < 0$ when $r''/(r')^2$ is sufficiently large. Thus, for a sufficiently large $-r''/(r')^2$ or α, there exists $\underline{b} < 1$ such that U_i is concave for all $b > \underline{b}$. Finally, $B_m \leq 2\eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)(\phi'(e_{im}))^2$ when $\phi''/(\phi')^2$ is sufficiently large. ■

Lemma EC.2. *There exists a unique vector $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$ that satisfies (4). Suppose that U_i is concave in $(e_{i1}, e_{i2}, ..., e_{iM})$. Then there exists a pure-strategy Nash equilibrium in the agent's subgame. Furthermore, a symmetric pure-strategy Nash equilibrium in this subgame solves (11)-(12).*

Proof. We first show that there exists a unique vector $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$ that solves (4). We first convert (4) into M equations, each of which consists of a single variable. From (4), we have $A_m\varphi(\widehat{e}_m)I_N = \eta'(\sum_{l=1}^M \phi(\widehat{e}_l))$ for all $m \in \{1,2,...,M\}$. Thus,

$$A_m\varphi(\widehat{e}_m) = A_l\varphi(\widehat{e}_l) \text{ for all } m,l \in \{1,2,...,M\},$$

where $\varphi(x) = (r'/\phi')(x)$. From this relationship, we obtain

$$\widehat{e}_l = \varphi^{-1}\left(\frac{A_m\varphi(\widehat{e}_m)}{A_l}\right). \tag{EC.3}$$

By plugging (EC.3) back into (4), we obtain

$$\Omega_m(\widehat{e}_m, A_1, A_2, ..., A_m) \equiv A_m r'(\widehat{e}_m)I_N - \eta'\left(\sum_{l=1}^M \phi\left(\varphi^{-1}\left(\frac{A_m\varphi(\widehat{e}_m)}{A_l}\right)\right)\right)\phi'(\widehat{e}_m) = 0. \tag{EC.4}$$

If (EC.4) has a solution for any $m \in \{1,2,...,M\}$, then (4) also has a solution. Let $\Phi_m(e) = \Omega_m(e, A_1, A_2, ..., A_m)$. Φ_m is continuous in e because r', η', ϕ, φ are continuous. Because r' is homogeneous of degree $-k$ and $(\eta' \circ \phi)\phi'$ is homogeneous of degree $(bp-1)$, we have either $\lim_{e\to 0} r'(e) = \infty$ and $\lim_{e\to\infty} r'(e) = 0$ or $\lim_{e\to 0}(\eta' \circ \phi)\phi' = 0$ and $\lim_{e\to\infty}(\eta' \circ \phi)\phi' = \infty$. Thus, we have $\lim_{e\to 0}\Phi_m(e) > 0$ and we have $\Phi_m(e) < 0$ for a sufficiently large e. Therefore, by the Intermediate Value Theorem, there exists $\widehat{e}_m$ such that $\Phi_m(\widehat{e}_m) = 0$.

We next show the uniqueness of $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$. Because η' is homogeneous of degree $(b-1)$ and letting $g = ((\eta \circ \phi)'/r')^{-1}$, we can rewrite (4) as follows:

$$\frac{A_m r'(\widehat{e}_m)I_N}{\eta'(\phi(\widehat{e}_m))\phi'(\widehat{e}_m)} = \frac{A_m I_N}{g^{-1}(\widehat{e}_m)} = \left(\frac{\phi(\widehat{e}_m)}{\sum_{l=1}^M \phi(\widehat{e}_l)}\right)^{1-b} \text{ for all } m \in \{1,2,...,M\}. \tag{EC.5}$$

Suppose to the contrary that there exists $(\widehat{\widehat{e}}_1, \widehat{\widehat{e}}_2, ..., \widehat{\widehat{e}}_M)$ that solves (EC.5) and is different than $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$. Then, for some m, $\widehat{\widehat{e}}_m \neq \widehat{e}_m$, and without loss of generality, suppose that $\widehat{\widehat{e}}_m > \widehat{e}_m$. As the left hand side of (EC.5) is decreasing in $\widehat{e}_m$ (since g^{-1} is increasing as assumed in §2), we need to have $\frac{\phi(\widehat{\widehat{e}}_m)}{\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l)} < \frac{\phi(\widehat{e}_m)}{\sum_{l=1}^M \phi(\widehat{e}_l)}$, which requires that $\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l) > \sum_{l=1}^M \phi(\widehat{e}_l)$. As $\sum_{m=1}^M \frac{\phi(\widehat{\widehat{e}}_m)}{\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l)} = \sum_{m=1}^M \frac{\phi(\widehat{e}_m)}{\sum_{l=1}^M \phi(\widehat{e}_l)} = 1$, there should exist $j \in \{1,2,...,M\}$ such that $\frac{\phi(\widehat{\widehat{e}}_j)}{\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l)} > \frac{\phi(\widehat{e}_j)}{\sum_{l=1}^M \phi(\widehat{e}_l)}$, which means that $\widehat{\widehat{e}}_j > \widehat{e}_j$ because $\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l) > \sum_{l=1}^M \phi(\widehat{e}_l)$. However, because $\frac{\phi(\widehat{\widehat{e}}_j)}{\sum_{l=1}^M \phi(\widehat{\widehat{e}}_l)} > \frac{\phi(\widehat{e}_j)}{\sum_{l=1}^M \phi(\widehat{e}_l)}$ and the left-hand side of (EC.5) is decreasing in $\widehat{e}_m$, we can deduce from (EC.5) that $\widehat{\widehat{e}}_j < \widehat{e}_j$. Thus, we have a contradiction. Therefore, there exists a unique $(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M)$ that solves (4).

We next show the existence of pure-strategy Nash equilibrium. According to Theorem 1.2 of Fudenberg and Tirole (1991), a pure-strategy Nash equilibrium among agents exists if each agent i's set of actions (i.e., the set of feasible efforts at different contests) is a non-empty, convex, and compact subset of the Euclidean space, and her utility U_i is continuous and quasi-concave in her effort e_i. Because each agent i has a capacity on her total effort $\overline{E}$, her action set can be restricted to $[0,\overline{E}]^M$, which is a non-empty, convex, and compact subset of the Euclidean space. Since U_i is continuous and concave (and hence quasi-concave), a pure-strategy Nash equilibrium exists.

Evaluating the agent's Kuhn-Tucker conditions at $e_{im} = e^*_m$ for all m yields (11)-(12). Because the agent's utility function U_i is concave, Kuhn-Tucker conditions are sufficient for optimality. Then, a solution to (11)-(12) yields the agent's best response, and hence it is a symmetric pure-strategy Nash equilibrium. Because the case where all agents exert zero efforts cannot be an equilibrium (since an agent can improve her utility by marginally increasing her effort), a symmetric pure-strategy Nash equilibrium should satisfy (11)-(12). ■

Lemma EC.3. *Suppose that all contests give the same award A and that U_i is concave in $(e_{i1}, e_{i2}, ..., e_{iM})$. There exists a unique solution to (11)-(12), which is the unique symmetric pure-strategy Nash equilibrium.*

Proof. Let λ be the Lagrange multiplier for the agent's capacity constraint. As shown in Lemma EC.2 of Online Appendix, a symmetric equilibrium satisfies the following Kuhn-Tucker conditions:

$$Ar'(e^*_m)I_N - \left(\frac{\phi(e^*_m)}{\sum_{l=1}^M \phi(e^*_l)}\right)^{1-b} \eta'\left(\phi(e^*_m)\right)\phi'(e^*_m) - \lambda^* = 0,\ m \in \{1,2,...,M\}. \tag{EC.6}$$

$$\lambda^*\left(\overline{E} - \sum_{m=1}^M e^*_m\right) = 0,\ \sum_{m=1}^M e^*_m \leq \overline{E},\ \text{and } e^*_m, \lambda^* \geq 0,\ m \in \{1,2,...,M\}. \tag{EC.7}$$

Let $\widehat{e} = g^{-1}(AI_N M^{1-b})$. Let $\Omega_m(e_1, e_2, ..., e_M) = Ar'(e_m)I_N - \left(\frac{\phi(e_m)}{\sum_{l=1}^M \phi(e_l)}\right)^{1-b} \eta'\left(\phi(e_m)\right)\phi'(e_m)$ for $m \in \{1,2,...,M\}$, so $\Omega_m(\widehat{e}, \widehat{e}, ..., \widehat{e}) = 0$ for all $m \in \{1,2,...,M\}$. Thus, by Lemma EC.2 of Online Appendix, $\widehat{e}$ is the unique solution to (EC.6) when $\lambda^* = 0$. When $M\widehat{e} < \overline{E}$, $(e^*_1, e^*_2, ..., e^*_M, \lambda^*) = (\widehat{e}, \widehat{e}, ..., \widehat{e}, 0)$ solves (EC.6)-(EC.7). Then, as shown in the proof of Lemma 1, $(e^*_1, e^*_2, ..., e^*_M, \lambda^*) = (\widehat{e}, \widehat{e}, ..., \widehat{e}, 0)$ is the unique symmetric equilibrium. When $M\widehat{e} > \overline{E}$, we show that $e^*_m = \overline{E}/M$ for all $m \in \{1,2,...,M\}$ solves (EC.6)-(EC.7) by verifying that in that case we can find $\lambda^* > 0$. Note that $\Omega_m(\overline{E}/M, \overline{E}/M, ..., \overline{E}/M) = \Omega_l(\overline{E}/M, \overline{E}/M, ..., \overline{E}/M)$ for all $m, l \in \{1,2,...,M\}$. Thus, because $\Omega_m(e, e, ..., e)$ is decreasing in e, and and $\widehat{e} > \overline{E}/M$, we have $\lambda^* = \Omega_m(\overline{E}/M, \overline{E}/M, ..., \overline{E}/M) > \Omega_m(\widehat{e}_1, \widehat{e}_2, ..., \widehat{e}_M) = 0$. Therefore, there exists $\lambda^* > 0$ such that $(\overline{E}/M, \overline{E}/M, ..., \overline{E}/M, \lambda^*)$ solves (EC.6)-(EC.7). Note that in this case $(\overline{E}/M, \overline{E}/M, ..., \overline{E}/M, \lambda^*)$ is the unique solution to (EC.6)-(EC.7) because for any $(e_1, e_2..., e_M)$ with $e_m > e_l$, we have $\Omega_m(e_1, e_2..., e_M) < \Omega_l(e_1, e_2..., e_M)$. ■

EC.2. Additional Results

Lemma EC.4. *(i) The cost function $\psi = \eta\left(\sum_{m=1}^M \phi(e_{im})\right)$ exhibits diseconomies of scale for each contest m; i.e., $\frac{\partial^2 \psi}{\partial e^2_{im}} \geq 0$ for all $m \in \{1,2,...,M\}$. When there is a single contest, i.e., $M = 1$, ψ is a convex function. (ii) ψ exhibits economies of scope across contests; i.e., $\frac{\partial^2 \psi}{\partial e_{im} e_{ij}} < 0$ for all $j \neq m$.*

Proof. (i) The partial derivative of ψ with respect to e_{im}

$$\frac{\partial\psi}{\partial e_{im}} = \eta'\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi'(e_{im}). \tag{EC.8}$$

Because η' is homogeneous of degree $(b-1)$, we have

$$\frac{\partial\psi}{\partial e_{im}} = \left(\frac{\sum_{l=1}^{M}\phi(e_{il})}{\phi(e_{im})}\right)^{b-1}\eta'\left(\phi(e_{im})\right)\phi'(e_{im}).$$

$\left(\frac{\sum_{l=1}^{M}\phi(e_{il})}{\phi(e_{im})}\right)^{b-1}$ is positive and increasing in e_{im} as $b<1$. Also, as $\eta\circ\phi$ is a convex function, $\eta'\left(\phi(e_{im})\right)\phi'(e_{im})$ is positive and increasing in e_{im}. Thus, $\frac{\partial\psi}{\partial e_{im}}$ is increasing in e_{im}, which means that $\frac{\partial^2\psi}{\partial e_{im}^2}>0$. When $M=1$, $\psi(e_{i1})=\eta\left(\phi(e_{i1})\right)$, which is convex because $\eta\circ\phi$ is convex by assumption.
(ii) Then, the cross partial derivative of ψ is

$$\frac{\partial^2\psi}{\partial e_{im}\partial e_{ij}} = \eta''\left(\sum_{l=1}^{M}\phi(e_{il})\right)\phi'(e_{im})\phi'(e_{ij}).$$

Because $\phi'>0$ and η is concave (i.e., $\eta''<0$), $\frac{\partial^2\psi}{\partial e_{im}\partial e_{ij}}<0$. ■

LEMMA EC.5. *In an optimal award scheme* $(A_1^*, A_2^*, ..., A_M^*)$ *that maximizes the average or total profit, there exist no contests* m *and* l *such that* $A_m^* > A_l^* > 0$.

Proof. For ease of illustration, we prove this result for two contests, but the proof can be extended to any number of contests. While we prove this result for the average profit objective, the same steps can be applied to prove the result for the total profit objective. Suppose to the contrary that it is optimal for the coordinator to give different awards at different contests. Without loss of generality, we label the contest with the largest award as contest 1 and the contest with the smallest award as contest 2. Then, in the optimal award scheme (A_1^*, A_2^*), $A_1^*>A_2^*$. Let e_1^* and e_2^* be the corresponding equilibrium effort at contest 1 and 2, respectively. It is never optimal to set an award such that the Lagrange multiplier λ in (11)-(12) is strictly positive because the average profit can be improved by marginally reducing awards. Thus, e_1^* and e_2^* should satisfy (EC.3), which means that $e_1^*>e_2^*$ because φ is decreasing. Consider a perturbation with an alternative set of awards (A_1, A_2) such that $r(e_1)=r(e_1^*)-\epsilon$ and $r(e_2)=r(e_2^*)+\epsilon$ (with a sufficiently small $\epsilon>0$ such that $\sum_{m=1}^{2}e_m\le\overline{E}$ due to the concavity of r). Because the total effort $\sum_{l=1}^{2}e_l\le\overline{E}$, we have $A_m=\frac{g^{-1}(e_m)}{I_N}\left(\frac{\phi(e_m)}{\sum_{l=1}^{2}\phi(e_l)}\right)^{1-b}$ from (4). Then, the change in the average profit $\overline{\Pi}$ after the perturbation is (note that $e_1^*=r^{-1}(r(e_1)+\epsilon)$, $e_2^*=r^{-1}(r(e_2)-\epsilon)$, $\sum_{m=1}^{2}r(e_m)=\sum_{m=1}^{2}r(e_m^*)$, and $E\left[\sum_{m=1}^{2}\widetilde{\xi}_{(1)m}^{N}\right]$ does not change after perturbation)

$$\begin{aligned}
\Delta &\equiv (-A_1+A_1^*-A_2+A_2^*)/2\\
&= -\frac{g^{-1}(e_1)}{2I_N}\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{1-b} + \frac{g^{-1}(e_1)}{2I_N}\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(r^{-1}(r(e_2)-\epsilon))}\right)^{1-b}\\
&\quad -\frac{g^{-1}(e_1)}{2I_N}\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(r^{-1}(r(e_2)-\epsilon))}\right)^{1-b} + \frac{g^{-1}(r^{-1}(r(e_1)+\epsilon))}{2I_N}\left(\frac{\phi(r^{-1}(r(e_1)+\epsilon))}{\phi(r^{-1}(r(e_1)+\epsilon))+\phi(r^{-1}(r(e_2)-\epsilon))}\right)^{1-b}\\
&\quad -\frac{g^{-1}(e_2)}{2I_N}\left(\frac{\phi(e_2)}{\phi(e_1)+\phi(e_2)}\right)^{1-b} + \frac{g^{-1}(e_2)}{2I_N}\left(\frac{\phi(e_2)}{\phi(r^{-1}(r(e_1)+\epsilon))+\phi(e_2)}\right)^{1-b}\\
&\quad -\frac{g^{-1}(e_2)}{2I_N}\left(\frac{\phi(e_2)}{\phi(r^{-1}(r(e_1)+\epsilon))+\phi(e_2)}\right)^{1-b} + \frac{g^{-1}(r^{-1}(r(e_2)-\epsilon))}{2I_N}\left(\frac{\phi(r^{-1}(r(e_2)-\epsilon))}{\phi(r^{-1}(r(e_1)+\epsilon))+\phi(r^{-1}(r(e_2)-\epsilon))}\right)^{1-b}.
\end{aligned}$$

Taking the limit $\lim_{\epsilon\to 0}\frac{2I_N\Delta}{\epsilon}$, and noting that $e_m = r^{-1}(r(e_m))$ and $\varphi(e_1^*)A_1^* = \varphi(e_2^*)A_2^*$, we obtain

$$\begin{aligned}\delta \equiv\ &(1-b)\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{-b}\frac{\phi(e_1)g^{-1}(e_1)}{(\phi(e_1)+\phi(e_2))^2}\frac{\phi'(e_2)}{r'(e_2)} - (1-b)\left(\frac{\phi(e_2)}{\phi(e_1)+\phi(e_2)}\right)^{-b}\frac{\phi(e_2)g^{-1}(e_2)}{(\phi(e_1)+\phi(e_2))^2}\frac{\phi'(e_1)}{r'(e_1)}\\ &+(1-b)\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{-b}\frac{\phi(e_2)g^{-1}(e_1)}{(\phi(e_1)+\phi(e_2))^2}\frac{\phi'(e_1)}{r'(e_1)} + \left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{1-b}\frac{1}{g'(g^{-1}(e_1))}\frac{1}{r'(e_1)}\\ &-(1-b)\left(\frac{\phi(e_2)}{\phi(e_1)+\phi(e_2)}\right)^{-b}\frac{\phi(e_1)g^{-1}(e_2)}{(\phi(e_1)+\phi(e_2))^2}\frac{\phi'(e_2)}{r'(e_2)} - \left(\frac{\phi(e_2)}{\phi(e_1)+\phi(e_2)}\right)^{1-b}\frac{1}{g'(g^{-1}(e_2))}\frac{1}{r'(e_2)}.\end{aligned}$$

Note that whenever $\delta > 0$, the average profit improves after the perturbation, so we prove that when k and b are sufficiently large, $\delta > 0$. Note that the first line in δ equals zero because $\phi'(e_m) = p\phi(e_m)/e_m$ and $g^{-1} = \eta'(\phi)\phi'/r'$. Furthermore, because $2-2k-bp \le 0$ (as assumed in §2),

$$\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{1-b}\frac{1}{g'(g^{-1}(e_1))}\frac{1}{r'(e_1)} > \left(\frac{\phi(e_2)}{\phi(e_1)+\phi(e_2)}\right)^{1-b}\frac{1}{g'(g^{-1}(e_2))}\frac{1}{r'(e_2)}. \tag{EC.9}$$

$\Upsilon(e_1,e_2) \equiv (1-b)\left(\frac{\phi(e_1)}{\phi(e_1)+\phi(e_2)}\right)^{-b}\frac{\phi(e_2)g^{-1}(e_1)}{(\phi(e_1)+\phi(e_2))^2}\frac{\phi'(e_1)}{r'(e_1)}$ approaches 0 as b approaches 1. Thus, when b is sufficiently close to 1, $\delta > 0$ from (EC.9). Furthermore, we have $\Upsilon(e_1,e_2) - \Upsilon(e_2,e_1) > 0$, and hence $\delta > 0$ whenever

$$\frac{\phi(e_1)^{-b}g^{-1}(e_1)}{\phi(e_1)}\frac{\phi'(e_1)}{r'(e_1)} > \frac{\phi(e_2)^{-b}g^{-1}(e_2)}{\phi(e_2)}\frac{\phi'(e_2)}{r'(e_2)}. \tag{EC.10}$$

As $\frac{\phi^{-b}g^{-1}}{\phi}\frac{\phi'}{r'}$ is homogeneous of degree $-bp+bp+k-1+p-1-p+k = 2k-2$, (EC.10) holds when $k \ge 1$. In either case, $\delta > 0$, which contradicts the optimality of $A_1^* > A_2^*$. ■

We next present five lemmas that are used in the proof of Theorem 1. The following two lemmas extend Lemmas EC.1 and EC.3 of Ales et al. (2017b) by changing the notation to fit to our paper and incorporating the agent's capacity constraint. Note that Ales et al. (2017b) assume $M = 1$.

LEMMA EC.6. *(Adopted from Lemma EC.1 of Ales et al. 2017b) Suppose that density h is log-concave. Then, $\mu_{(j)}^{N+1} - \mu_{(j)}^N < \mu_{(j+1)}^{N+1} - \mu_{(j+1)}^N$ for all $j \in \{1,2,...,N-1\}$.*

Proof. Let $\delta_{(j)}^N \equiv \mu_{(j)}^N - \mu_{(j+1)}^N$. We want to show that $\delta_{(j)}^{N+1} < \delta_{(j)}^N$ for all j. From Galton (1902),

$$\delta_{(j)}^N = \binom{N}{j}\int_{\underline{s}}^{\overline{s}} H(s)^{N-j}(1-H(s))^j ds.$$

Rewriting this equation in terms of $h_{(j)}^N(s)$, and integrating it by parts, we obtain

$$\delta_{(j)}^N = \frac{1}{j}\int_{\underline{s}}^{\overline{s}} h_{(j)}^N(s)\frac{(1-H(s))}{h(s)}ds = \frac{1}{j}H_{(j)}^N(s)\frac{(1-H(s))}{h(s)}\Big|_{\underline{s}}^{\overline{s}} - \frac{1}{j}\int_{\underline{s}}^{\overline{s}} H_{(j)}^N(s)\left(\frac{(1-H(s))}{h(s)}\right)' ds.$$

Using the equation above, we can derive $\delta_{(j)}^{N+1} - \delta_{(j)}^N$ as

$$\delta_{(j)}^{N+1} - \delta_{(j)}^N = \mu_{(j)}^{N+1} - \mu_{(j)}^N - (\mu_{(j+1)}^{N+1} - \mu_{(j+1)}^N) = \int_{\underline{s}}^{\overline{s}}[H_{(j)}^N(s) - H_{(j)}^{N+1}(s)]\left(\frac{(1-H(s))}{h(s)}\right)' ds < 0,$$

because $H_{(j)}^{N+1}(s) \le H_{(j)}^N(s)$ for all s (and $H_{(j)}^{N+1}(s) < H_{(j)}^N(s)$ for a measurable subset of Ξ), and log-concavity implies that $\left(\frac{(1-H(s))}{h(s)}\right)' < 0$ for all s. ■

LEMMA EC.7. *(Adopted from Lemma EC.3 of Ales et al. 2017b) Suppose that $M = 1$, and that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. Then, $\lim_{\alpha\to\infty}\frac{A^*}{\alpha} = 0$.*

Proof. When $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with $\alpha > 0$, I_N is converted to $\widehat{I}_N = I_N/\alpha$. Note that when $M = 1$, relaxing the agent's capacity constraint, the optimal award $\widehat{A}[\alpha]$ satisfies

$$r'\left(g\left(\frac{\widehat{A}[\alpha]I_N}{\alpha}\right)\right)g'\left(\frac{\widehat{A}[\alpha]I_N}{\alpha}\right)\frac{I_N}{\alpha} - 1 = 0. \tag{EC.11}$$

Because $r'(g(x))g'(x)$ is decreasing in x, and I_N/α is decreasing in α, for $\widehat{A}[\alpha]$ to satisfy (EC.11), $\widehat{A}[\alpha]/\alpha$ should be decreasing in α. Since $\widehat{A}[\alpha]/\alpha$ is decreasing in α, and $\widehat{A}[\alpha] \geq 0$, $\widehat{A}[\alpha]/\alpha$ converges. Furthermore, because $\lim_{\alpha\to\infty} \frac{I_N}{\alpha} = 0$, we need $\lim_{\alpha\to\infty} \frac{\widehat{A}[\alpha]}{\alpha} = 0$ to satisfy (EC.11). Under $\widehat{A}$, the equilibrium effort $e^* = g\left(\frac{\widehat{A}[\alpha]I_N}{\alpha}\right)$. Because $\lim_{\alpha\to\infty} \frac{\widehat{A}[\alpha]}{\alpha} = 0$, for a sufficiently large α, we have $e^* \leq \overline{E}$, so $A^* = \widehat{A}$. Thus, $\lim_{\alpha\to\infty} \frac{A^*[\alpha]}{\alpha} = 0$. ■

LEMMA EC.8. *For any l, n_1, and n_2 ($\in \mathbb{Z}_{++}$) such that $n_2 > n_1$, we have $\mu_{(1)}^{n_2+l} - \mu_{(1)}^{n_2} < \mu_{(1)}^{n_1+l} - \mu_{(1)}^{n_1}$.*

Proof. We first show that $\mu_{(1)}^{n_1+2} - \mu_{(1)}^{n_1+1} < \mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$. According to Relation 1 on page 44 of David and Nagaraja (2003), $(n_1+1)\mu_{(1)}^{n_1} = n\mu_{(1)}^{n_1+1} + \mu_{(2)}^{n_1+1}$. From this relationship, we can show that $\mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1} = \frac{1}{n_1+1}(\mu_{(1)}^{n_1+1} - \mu_{(2)}^{n_1+1})$. Lemma EC.6 shows that $(\mu_{(1)}^{n_1+1} - \mu_{(2)}^{n_1+1}) > 0$ and is decreasing in n_1. Because $\frac{1}{n_1+1}$ is also decreasing in n_1, so $\mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$ is decreasing in n_1. We use induction for the rest of the proof. From the above discussion, we have $\mu_{(1)}^{n_1+j+2} - \mu_{(1)}^{n_1+j+1} < \mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$. Suppose that for $l > 1$, we have $\mu_{(1)}^{n_1+l} - \mu_{(1)}^{n_1+l-1} < \mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$. Then, because $\mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$ is decreasing in n_1, $\mu_{(1)}^{n_1+l+1} - \mu_{(1)}^{n_1+l} < \mu_{(1)}^{n_1+l} - \mu_{(1)}^{n_1+l-1} < \mu_{(1)}^{n_1+1} - \mu_{(1)}^{n_1}$. Thus, we can rewrite this relationship as $\mu_{(1)}^{n_1+l+1} - \mu_{(1)}^{n_1+1} < \mu_{(1)}^{n_1+l} - \mu_{(1)}^{n_1}$. Using an induction as above, we can show that for any $n_2 > n_1$, we have $\mu_{(1)}^{n_2+l} - \mu_{(1)}^{n_2} < \mu_{(1)}^{n_1+l} - \mu_{(1)}^{n_1}$. ■

LEMMA EC.9. *Let $N_1 = \lfloor N/2 \rfloor$ and $N_2 = \lceil N/2 \rceil$. $\mu_{(1)}^{N_1} + \mu_{(1)}^{N_2} > \mu_{(1)}^{N_1'} + \mu_{(1)}^{N_2'}$ for any $N_1' \in \{1, 2, .., N\} \setminus \{N_1, N_2\}$ and $N_2' = N - N_1'$.*

Proof. Suppose without loss of generality that $N_2' > N_1'$ ($N_1' = N_2'$ is not possible as $N_1' \notin \{N_1, N_2\}$). Then, we have $N_2' > N_2 \geq N_1 > N_1'$, and $l \equiv N_2' - N_2 = N_1 - N_1' > 0$. As $N_1' < N_2$, by letting $n_1 = N_1'$ and $n_2 = N_2$, by Lemma EC.8, $\mu_{(1)}^{N_1'+l} - \mu_{(1)}^{N_1'} > \mu_{(1)}^{N_2+l} - \mu_{(1)}^{N_2}$. Thus, $\mu_{(1)}^{N_1} + \mu_{(1)}^{N_2} > \mu_{(1)}^{N_1'} + \mu_{(1)}^{N_2'}$. ■

LEMMA EC.10. *Suppose that the are two exclusive contests. There exists $\alpha_e > 0$ such that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha\widetilde{\xi}_{im}$ with a scale parameter $\alpha > \alpha_e$, it is optimal for the coordinator to distribute the number of agents as evenly as possible between contests, where one contest has $\lfloor N/2 \rfloor$ agents and the other has $\lceil N/2 \rceil$ agents.*

Proof. Suppose to the contrary that it is not optimal for the coordinator to distribute agents as proposed. Instead, suppose that it is optimal for the coordinator to assign N_1 agents to the first contest and N_2 agents to the second contest. Let A_1 and A_2 be the optimal award in the first and second contest, respectively. Note that we have $N_1 \notin \{\lfloor N/2 \rfloor, \lceil N/2 \rceil\}$ and $N_2 = N - N_1$. Because agents are split between contests, each contest is an individual contest with different set of agents, so the equilibrium effort can be derived from (4) under $M=1$. Thus, $e_m^* = g(A_m I_{N_m})$. Then, the average profit can be written as $\overline{\Pi} = \frac{1}{2}\sum_{m=1}^{2}\left(r(e_m^*) + \mu_{(1)}^{N_m} - A_m\right)$. Consider the alternative case where the coordinator assigns $\lfloor N/2 \rfloor$ agents to the first contest and $\lceil N/2 \rceil$ agents to the second contest, and gives awards so that agents' efforts are the same at each contest. Specifically, the award in the first contest is $\frac{A_1 I_{N_1}}{I_{\lfloor N/2 \rfloor}}$, and the award in the second contest is $\frac{A_2 I_{N_2}}{I_{\lceil N/2 \rceil}}$. Also, suppose that the output shock $\widetilde{\xi}_{im}$ is transformed to $\widehat{\xi}_{im} = \alpha \widetilde{\xi}_{im}$ with a scale parameter $\alpha > 0$. After the transformation, $E[\widehat{\xi}_{im}] = \alpha E[\widetilde{\xi}_{im}]$ and $\widehat{I}_N = I_N/\alpha$ for all N. In this case, when the number of agents at each contest changes from N_1 to $\lfloor N/2 \rfloor$ and N_2 to $\lceil N/2 \rceil$, and the awards are given as discussed above, the change in the average profit can be written as

$$\Delta\overline{\Pi} \equiv \frac{\alpha}{2}\left(\mu_{(1)}^{\lfloor N/2 \rfloor} + \mu_{(1)}^{\lceil N/2 \rceil} - \sum_{m=1}^{2}\mu_{(1)}^{N_m} + \frac{A_1}{\alpha}\left(1 - \frac{I_{N_1}}{I_{\lfloor N/2 \rfloor}}\right) + \frac{A_2}{\alpha}\left(1 - \frac{I_{N_2}}{I_{\lceil N/2 \rceil}}\right)\right). \tag{EC.12}$$

As Lemma EC.9 shows, $\left(\mu_{(1)}^{\lfloor N/2 \rfloor} + \mu_{(1)}^{\lceil N/2 \rceil} - \sum_{m=1}^{2}\mu_{(1)}^{N_m}\right) > 0$ for $m \in \{1,2\}$, and as Lemma EC.7 shows, when $r'(g(x))g'(x)$ is decreasing in x, $\lim_{\alpha\to\infty} A_m^*/\alpha = 0$ for each m. Thus, for a sufficiently large α, we have $\Delta\overline{\Pi} > 0$, which contradicts the optimality of N_1 and N_2. Because there is a finite number of combinations for N_1 and N_2, there exists α_e such that for any $\alpha > \alpha_e$, $\Delta\overline{\Pi} > 0$ for any N_1 and N_2. Thus, when $\alpha > \alpha_e$, it is optimal for the coordinator to run one contest with $\lfloor N/2 \rfloor$ agents and the other contest with $\lceil N/2 \rceil$ agents. ■