Persuasion Meets Delegation

Anton Kolotilin (UNSW) and Andy Zapechelnyuk (St Andrews)

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Principal – Agent Problem

- A principal wants to influence the decision of a biased agent

- Two instruments of influence
  - Delegation
  - Persuasion

- How are they related?

Preview of Main Result

- Two problems are equivalent under general assumptions
  - Explicit equivalence mapping between the two problems
  - Decisions and states are swapped in the two problems

Who Cares?

- Results in one problem to solve the other problem

- Reinterpretations of insights in the two problems

- Stepping stone for relations in other problems and extensions

Outline

- Persuasion and delegation problems

- Equivalence result

- Application to monopoly regulation

A Problem

- Principal (she) and Agent (he)

- Agent makes a decision $x \in [0, 1]$

- State $\theta \in [0, 1]$ is uniformly distributed

Payoffs

- Agent's and Principal's payoffs are $U(\theta, x)$ and $V(\theta, x)$

- $\frac{\partial}{\partial x}U(\theta, x)$ and $\frac{\partial}{\partial x}V(\theta, x)$ are continuous in θ and x

- $\frac{\partial}{\partial x}U(\theta, x)$ is strictly increasing in θ and strictly decreasing in x

- A pair (U, V) is called a *primitive*

- $\mathcal{P}$ is the set of primitives that satisfy the above assumptions

Monotone Persuasion Problem

- Principal chooses a monotone partition Π
  - Π is the union of separated states $\{\theta\}$
  - Π^c is the union of pooling intervals (θ', θ'')

- Π is a closed subset of $[0, 1]$ that contains 0 and 1

- Denote by $\mathbf{\Pi}$ the set of all such Π

Why Monotone Persuasion?

- Monotone partitions are widespread:
  - Credit ratings of financial institutions
  - Consumer ratings of services on Amazon, Yelp,...
  - Grade conversion schemes from 100-point to ABC scale
- Conditions for optimality of monotone partitions: Dworczak-Martini (2019)
- Characterization of optimal monotone partitions: Kolotilin and Li (2019)

Monotone Persuasion Problem

- Denote by $\mu_\Pi(\theta)$ the partition element that contains θ
  - Interpret $\mu_\Pi(\theta)$ as a message sent at state θ

- After observing $\mu_\Pi(\theta)$, Agent chooses

$$x_P^*(\theta, \Pi) \in \arg\max_{x \in [0,1]} \mathbb{E}[U_P(s, x) \mid s \in \mu_\Pi(\theta)]$$

- Principal's problem:

$$\max_{\Pi \in \mathbf{\Pi}} \mathbb{E}[V_P(\theta, x_P^*(\theta, \Pi))]$$

Balanced Delegation Problem

- Principal chooses a set $\Pi \in \mathbf{\Pi}$ of decisions
  - Interpret Π as a *delegation set*

- After privately observing θ, Agent chooses

$$x_D^*(\theta, \Pi) \in \arg\max_{x \in \Pi} U_D(\theta, x)$$

- Principal's problem:

$$\max_{\Pi \in \mathbf{\Pi}} \mathbb{E}[V_D(\theta, x_D^*(\theta, \Pi))]$$

Why Balanced Delegation?

- A balanced delegation set Π must contain $\{0, 1\}$

- A *balanced delegation problem* is a delegation problem with extra boundary conditions, which includes:
  - Standard delegation problems under general assumptions
  - Novel delegation problems with participation constraints

Persuasion versus Delegation

What is the difference between these problems?

Main Result

The monotone persuasion problem and the balanced delegation problem are "equivalent".

Definition

Primitives (U_P, V_P) and (U_D, V_D) are equivalent if

$$\mathbb{E}\Big[V_P(\theta, x_P^*(\theta, \Pi))\Big] = \mathbb{E}\Big[V_D(\theta, x_D^*(\theta; \Pi))\Big] \quad \text{for all } \Pi \in \mathbf{\Pi}.$$

Theorem

For each $(U_D, V_D) \in \mathcal{P}$, an equivalent $(U_P, V_P) \in \mathcal{P}$ is

$$U_P(\theta, x) = -\int_0^x \left.\frac{\partial U_D(t,s)}{\partial s}\right|_{s=\theta} \mathrm{d}t, \quad V_P(\theta, x) = -\int_0^x \left.\frac{\partial V_D(t,s)}{\partial s}\right|_{s=\theta} \mathrm{d}t.$$

Conversely, for each $(U_P, V_P) \in \mathcal{P}$, an equivalent $(U_D, V_D) \in \mathcal{P}$ is

$$U_D(\theta, x) = \int_x^1 \left.\frac{\partial U_P(s,t)}{\partial t}\right|_{t=\theta} \mathrm{d}s, \quad V_D(\theta, x) = \int_x^1 \left.\frac{\partial V_P(s,t)}{\partial t}\right|_{t=\theta} \mathrm{d}s.$$

$$\left.\frac{\partial U_D(t,x)}{\partial x}\right|_{x=s} + \left.\frac{\partial U_P(s,x)}{\partial x}\right|_{x=t} = 0$$

and

$$\left.\frac{\partial V_D(t,x)}{\partial x}\right|_{x=s} + \left.\frac{\partial V_P(s,x)}{\partial x}\right|_{x=t} = 0$$

Tractable Persuasion and Delegation Problems

- *Linear Persuasion*, as in Kamenica and Gentzkow (2011):

$$\frac{\partial U_P(\theta, x)}{\partial x} = c(\theta) - b(x) \quad \text{and} \quad \frac{\partial V_P(\theta, x)}{\partial x} = c(\theta) - d(x),$$

  where b and c are continuous and strictly increasing.

- *Linear Delegation*, as in Amador and Bagwell (2013):

$$\frac{\partial U_D(\theta, x)}{\partial x} = b(\theta) - c(x) \quad \text{and} \quad \frac{\partial V_D(\theta, x)}{\partial x} = d(\theta) - c(x),$$

  where b and c are continuous and strictly increasing.

- Linear Persuasion and Linear Delegation are equivalent

Application: Monopoly Regulation

- x and q denote price and quantity
- Linear demand function: $q = 1 - x$
- Linear cost function cq, where $c \in [0, 1]$ is a private cost
- Marginal cost c has a positive **unimodal** density f
- Profit and welfare are given by

  $U(c, x) = (x - c)(1 - x)$ and $V(c, x) = U(c, x) + \frac{1}{2}(1 - x)^2$

- Regulator chooses a set $\Pi \subset [0, 1]$ of prices available for Monopolist, and Monopolist maximizes profit

Application: Monopoly Regulation

- Two versions:
  - Regulation without Monopolist's participation constraint (studied by Alonso and Matouschek 2008)
  - Regulation with Monopolist's participation constraint (studied by Amador and Bagwell 2019)

Participation Constraint

- Monopolist can always choose to produce zero quantity, equivalently set price $x = 1$, so $1 \in \Pi$

- Selling at zero price is less profitable than not producing, so, w.l.o.g., $0 \in \Pi$

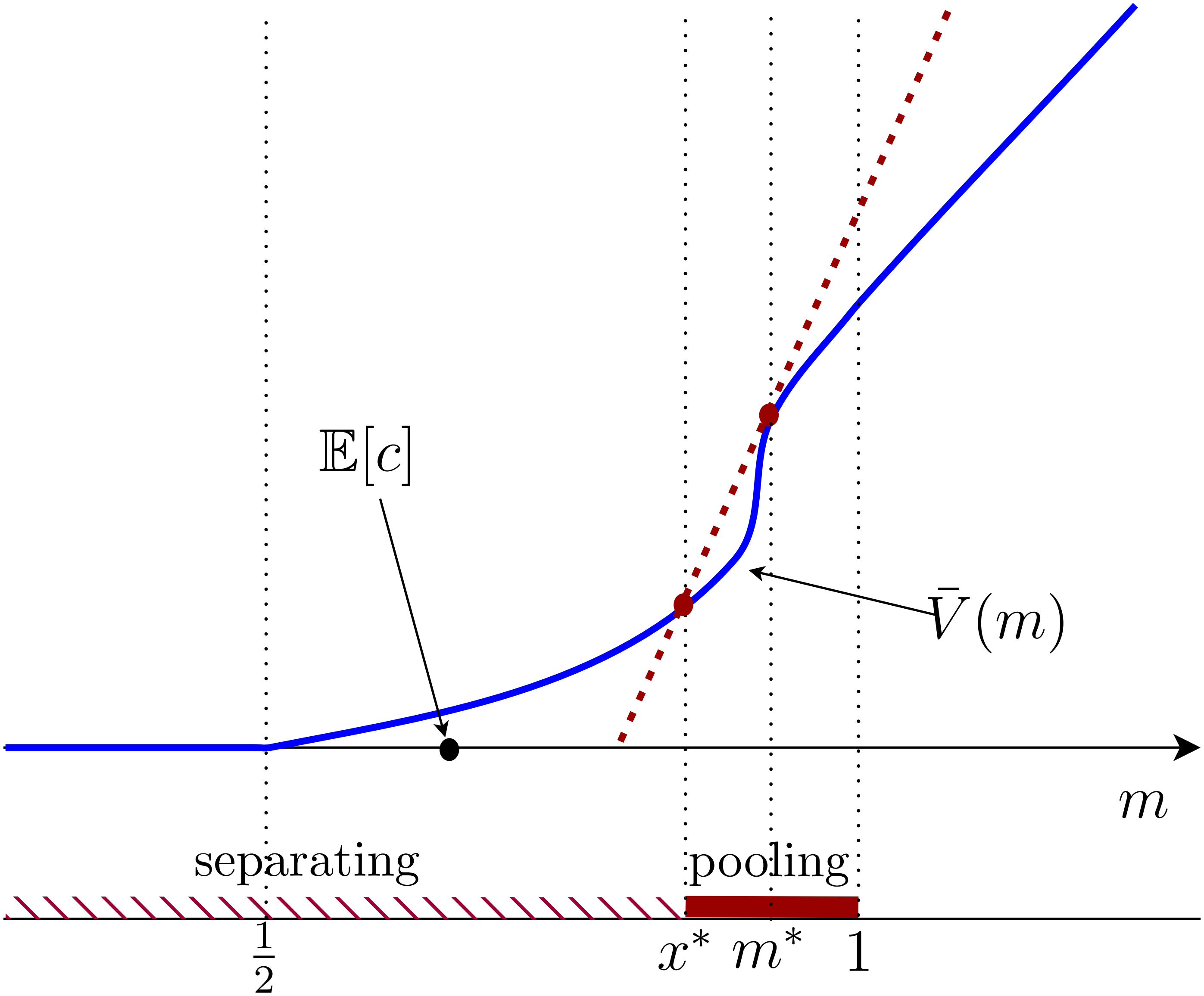
- Defining $\theta = F(c)$ yields a balanced delegation problem

Equivalent Persuasion Problem

Principal's payoff from a message $\mu_\Pi(\theta)$ is

$$\bar{V}(m) = \int_0^{2m-1} (m - c) \mathrm{d}F(c),$$

where $m = \mathbb{E}[s|s \in \mu_\Pi(\theta)]$ and $\theta \sim U[0, 1]$.



$\mathbb{E}[c]$
$\bar{V}(m)$
m
separating
pooling
$\frac{1}{2}$
x^*
m^*
1

Solution

- Under unimodal f, $\Pi = [0, x^*] \cup \{1\}$ is optimal

- *Upper censorship* in the persuasion problem

- *Price cap* in the regulation problem

Regulation without Participation Constraint

- Extend profit U_D and welfare V_D to the domain $[0, 2]$ of prices

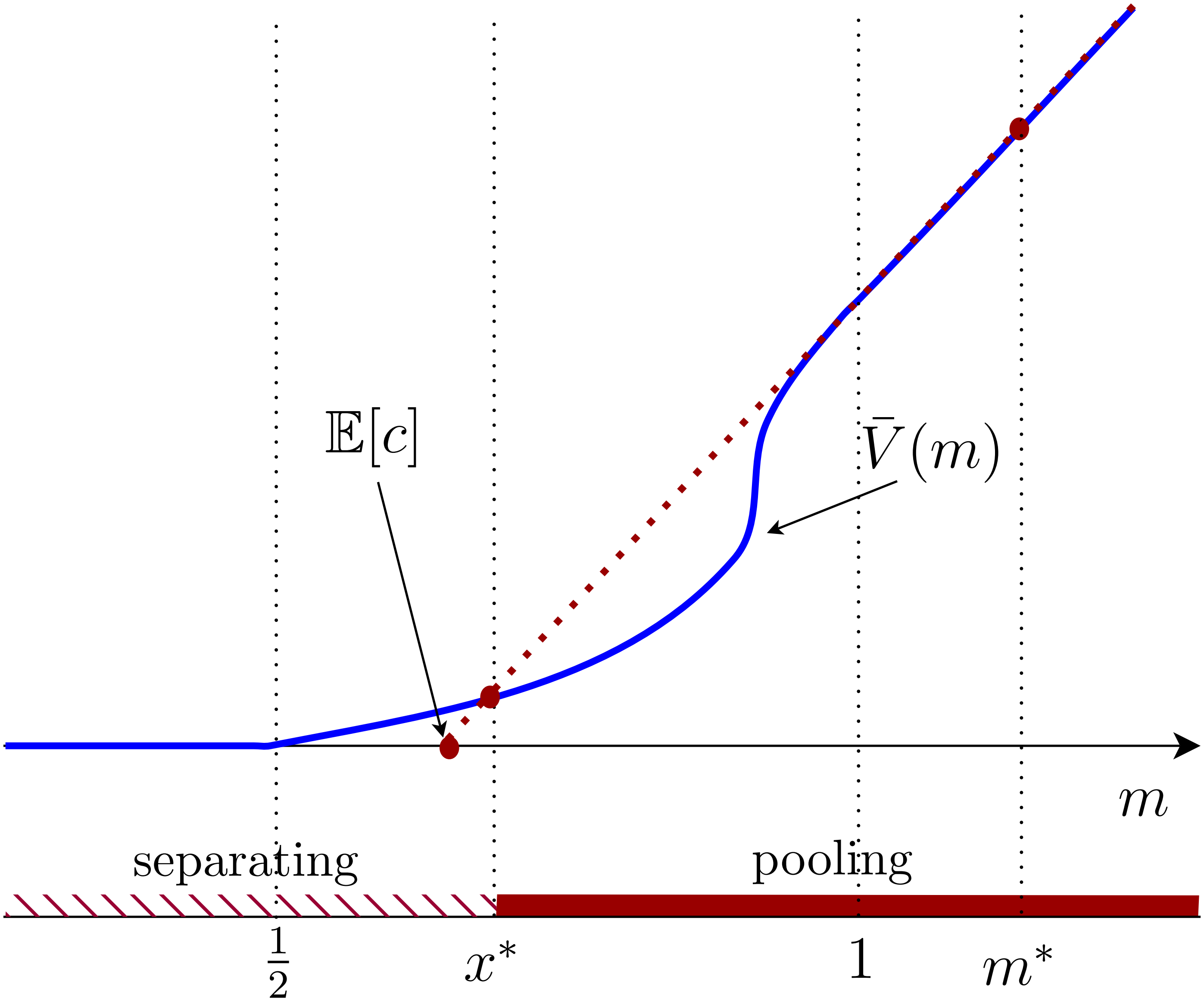
- **Lemma.** If $\Pi \subset [0, 2]$ is optimal, then $\Pi \cup \{0, 2\}$ is optimal.

Equivalent Persuasion Problem

Principal's payoff from a message $\mu_{\Pi}(\theta)$ is

$$\bar{V}(m) = \int_0^{2m-1} (m - c)\mathrm{d}F(c),$$

where $m = \mathbb{E}[s|s \in \mu_{\Pi}(\theta)]$ and $\theta \sim U[0, 2]$.



$\mathbb{E}[c]$
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m
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x^*
1
m^*

Discussion

- Monopoly regulation with and without participation constraint is solved using a single result from the persuasion literature
- Price cap is optimal in both versions of the problem
- Price cap is higher with the participation constrained

Conclusion

- The monotone persuasion problem and the balanced delegation problem are equivalent

- Insights and results in one problem can be used to understand and solve the other problem

- Novel delegation problems with participation constraints and new results for standard delegation problems