Strategy-proof and Efficient Mediation: An Ordinal Mechanism Design Approach

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Alternative Dispute Resolution: Mediation

- Dispute: disagreement between two (or more) parties about issues that are important for them

- **Mediation** is a voluntary process: self-determination

- Neutral and impartial third party facilitates communication and negotiation; promotes exploration of mutually acceptable alternatives

- *Unlike litigation and arbitration, mediation doesn't search for truth, rather searches for satisfaction*

- The emphasis is not on who is right or wrong, but rather upon establishing a workable solution that meets the participants' needs

Mediation: a growth industry

- Face to Face Mediation:
  - Courts in all US states offer some form of ADR
  - 17 states require mandatory mediation
  - Court-connected mediation numbers
    - 35.6% of civil and 21.6% of divorce cases in NY courts (2016)
    - 11% of civil cases in Northern California (2011)
    - 15-20% of civil cases in Norway/Finland (2015)
  - Total value of mediated cases in UK is £10.5bn., mega-cases excluded, (25% of UK defense budget)
- Online Dispute Resolution: Small disputes but large in number
  - Disputes arising online
  - One billion disputes/year in E-bay, paypal, Uber, Amazon
  - Automated negotiation mechanisms (e.g., "split the difference")
- Data suggests
  - 60-90% success rate (higher if partial resolutions are included)
  - 90-95% satisfaction; higher rate of compliance relative to court orders

Cons of Mediation

- Often considered less formal and less transparent than adjudication

> *"The competitive presentation of evidence in the formal adversarial system as counteracts decision maker bias and produces fairer and more accurate decisions than less formal systems."*
> Damaska (1975)

- LaFree and Rack (1996) provide empirical evidence that white males receive significantly more favorable outcomes in mediation relative to minority females.
- Tyler and Huo (2002) advocate for the use of fair procedures in which decisions are viewed as *neutral, objective, and consistent.*
- Question: *Can we make mediatition a more structured and rigorous process without compromising the mediator's primary role?*

Main Question

- *Is there an impartial and incentive compatible way to soliciting true preferences so that final outcomes are efficient and individually rational?*
- Malhotra and Bazerman, 2007: Success in mediation/negotiation lies in parties' ability of expanding the pie and finding integrative (win-win) outcomes: **multi-issue** and **logrolling**
  - *Logrolling*: You offer the other side something that they value more than you, in exchange for gaining something from them that you value more than they do

Essentials of the model

- *Assisted Negotiation*: modeled, without loss of generality, as a mechanism design problem,

- *Ordinal approach*: Ordinal rankings over alternatives; possibly multiple issues; allowing for "utility and prior free" analysis in line with Wilson doctrine (robust mech. design),

> *"... Despite these virtues, mechanism design has two weaknesses. First, the mechanisms depend in complex ways on the traders' beliefs and utility functions, which are assumed to be common knowledge. Second, it allows too much commitment. In practice, bargainers use simple trading rules—such as a sequence of offers and counteroffers—that do not depend on beliefs or utility functions. "*
> Handbook of Game Theory (Ausubel, Crampton & Deneceker)

- *Private information*: about acceptable alternatives, requiring individual rationality; mechanism design without commitment

Gains from Mediation: Two Types of Issues

- **Type X:** An issue has *uncertain gains from mediation* if it is unknown that a mutually agreeable resolution exists
- **Type Y:** An issue has *certain gains from mediation* if it is commonly known that mutually agreeable resolutions exist
- **Example:** Consider an employment negotiation between a company and a prospective employee. There are two issues up for negotiation: job title and compensation.
  - *Job Title*: X= {Engineer, Project leader, Manager, Senior Manager}
  - *Compensation*: Y= [$100K, $250K]
- **Importance:** An issue of type-X creates an "incentive problem" while an issue of type-Y creates a pure "fair division" problem
- *All issues are solvable via strategy-proof and efficient mechanisms if only if they are considered jointly*

A simple example: single-issue mediation

- Negotiators 1 and 2 are in a dispute over a single issue X with uncertain gains from mediation

- $X = \{x_1, x_2, o\}$

- The **outside option** (o): outcome in case agents can't reach an agreement via mediation

- Commonly known: 1 prefers x_1 to x_2, and 2 prefers x_2 to x_1

- Ranking of o is private information.

Example

- Types of the negotiators:

$\theta_1^{x_1}$	$\theta_1^{x_2}$
x_1	x_1
o	x_2
x_2	o

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- If mediation is a mechanism that maps private information to an outcome,

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- Thus, no efficient, individually rational, strategy-proof mediation !

The Main Model:
Multi-issue mediation

- Negotiators 1 and 2 are in dispute over **two issues**: X and Y
    - $X = \{x_1, ..., x_m, o_X\}$ and
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- **Public Info:** $\forall\, i, 1 \leq k \leq m-1$ and all $\theta_i^Z \in \Theta_i^Z$
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  - Inefficient outside option in Y: $y \; \theta_i^Y \; o_Y$ for all $y \in Y \setminus \{o_Y\}$

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- Type space for i: $\Theta_i = \Theta_i^X$
- Set of type profiles: $\Theta = \Theta_1 \times \Theta_2$

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- Such an equilibrium exists iff f is **strategy-proof** and **(ex-post) individually rational**

Admissible preferences over bundles

- Efficiency, SP, IR require preferences over bundles; *domain restriction*
- A **bundle** $b = (x, y) \in X \times Y$
- $\Re$: set of all complete and transitive binary relations over bundles
- $b \; R \; b'$ means "bundle b is at least as good as b'.
- P is the strict counterpart of R
- An **extension map** $\Lambda : \Theta_i \to \Re$

Admissible preferences over bundles

- Let $A(\theta_i) = \{x \in X | x \; \theta_i \; o_X\}$ be the set of acceptable alternatives
- The extension map Λ is **regular** if the followings hold for all i, θ_i, $R_i \in \Lambda(\theta_i)$:

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   1. [monotonicity] for any $x, x' \in X$ and $y, y' \in Y$ with $(x,y) \neq (x',y')$

   $$(x,y) \; P_i \; (x',y')$$

   whenever $\left[x \; \theta_i^X \; x' \text{ or } x = x'\right]$ and $\left[y \; \theta_i^Y \; y' \text{ or } y = y'\right]$

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2. [Deal breakers] for any $y, y' \in Y \setminus \{o_Y\}$,

$$(x, y) \; R_i \; (x', y')$$

   whenever $x \in A(\theta_i) \cup \{o_X\}$, $x' \notin A(\theta_i)$, and $x \neq x'$

Definitions

- The mediation rule f is **strategy-proof** if for all i and all $\theta_i \in \Theta_i$,

$$f(\theta_i, \theta_{-i}) \; R_i \; f(\theta_i', \theta_{-i})$$

  for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$ and all $\theta_{-i} \in \Theta_{-i}$

- f is **individually rational** if for all i and all $(\theta_i, \theta_{-i}) \in \Theta$,

$$f(\theta_i, \theta_{-i}) \; R_i \; (o_X, o_Y)$$

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- f is (Pareto) **efficient** if there exists no $(\theta_i, \theta_{-i}) \in \Theta$ and $(x', y') \in X \times Y$ such that

$$(x', y') \; R_i \; f(\theta_i, \theta_{-i})$$

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Main Results

The Possibility Result: Partial characterization

$f =$

	$\theta_2^{x_1}$	$\cdots$	$\theta_2^{x_m}$
$\theta_1^{x_1}$	$f_{1,1}$	$\cdots$	$f_{1,m}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\theta_1^{x_m}$	$f_{m,1}$	$\cdots$	$f_{m,m}$

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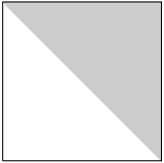

Theorem 1

The mediation rule f is efficient, IR and SP only if the following hold:

(i) *If $\ell < j$, then $f_{\ell,j} = (o_X, y)$ for some $y \in Y$*

(ii) *If $\ell = j$, then $f_{\ell,j} = (x_\ell, \hat{y}_\ell)$ where $\hat{y}_\ell = y_{m+1-\ell} \in Y$*

(iii) *(Adjacency) If $\ell > j$, then $f_{\ell,j} \in \{f_{\ell-1,j}, f_{\ell,j+1}\} \subset \mathbf{B} = \{(x_\ell, \hat{y}_\ell) | \ell = 1, ..., m\}$ and there exists strict precedence order $\triangleright$ on $\mathbf{B}$ such that*

$$f_{\ell,j} = \begin{cases} f_{\ell-1,j}, & \text{if } f_{\ell-1,j} \triangleright f_{\ell,j+1} \\ f_{\ell,j+1}, & \text{oth.} \end{cases}$$

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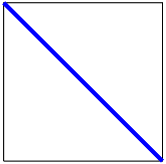

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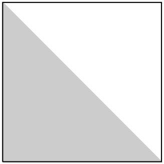
A partial characterization



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Example: A member of the adjacent rules family

$6 \triangleright 7 \triangleright 14 \triangleright 2 \triangleright 5 \triangleright 10 \triangleright 12 \triangleright 9 \triangleright 15 \triangleright 1 \triangleright 4 \triangleright 3 \triangleright 8 \triangleright 11 \triangleright 13$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
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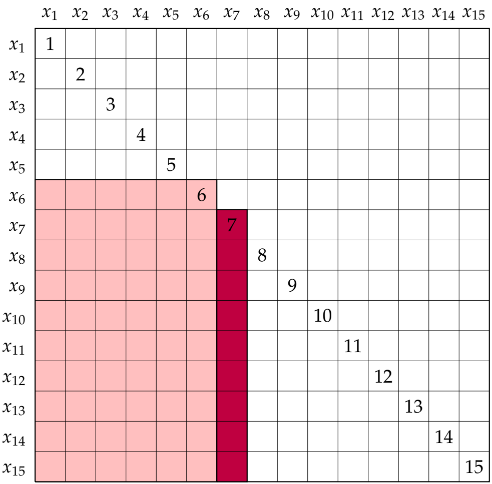

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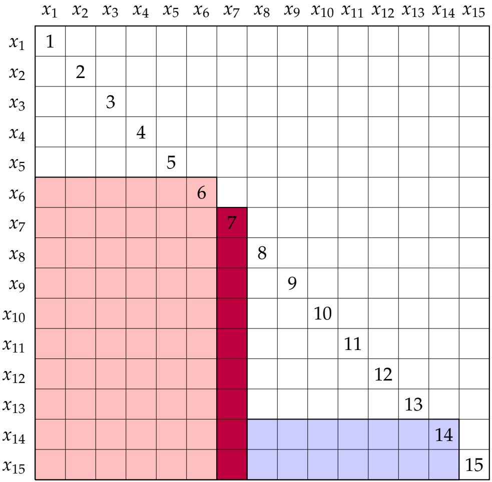
$6 \triangleright 7 \triangleright 14 \triangleright 2 \triangleright 5 \triangleright 10 \triangleright 12 \triangleright 9 \triangleright 15 \triangleright 1 \triangleright 4 \triangleright 3 \triangleright 8 \triangleright 11 \triangleright 13$



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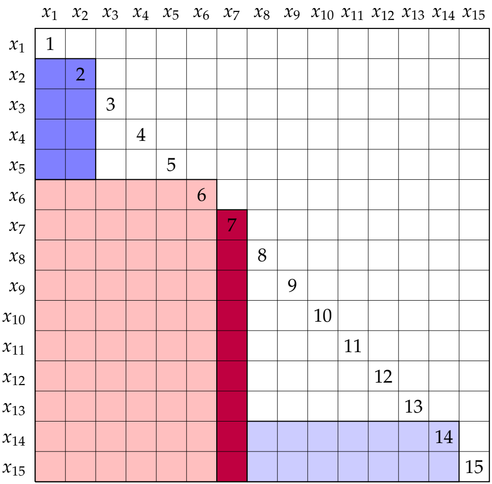

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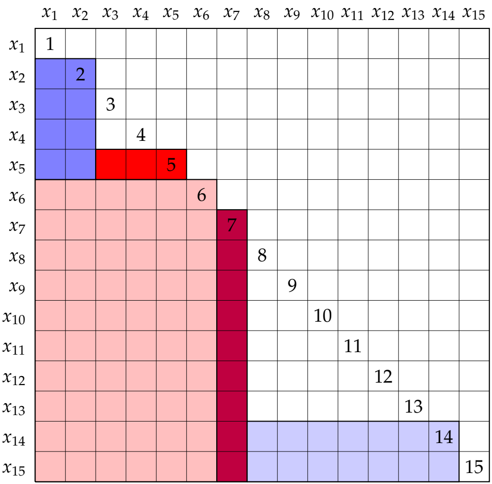

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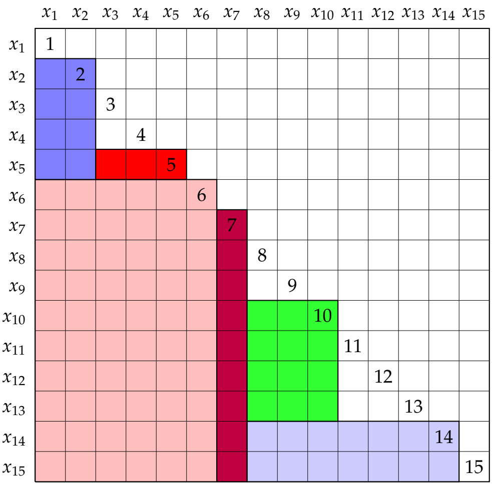

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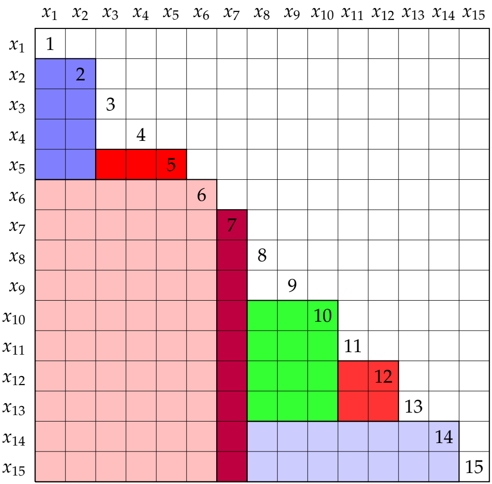

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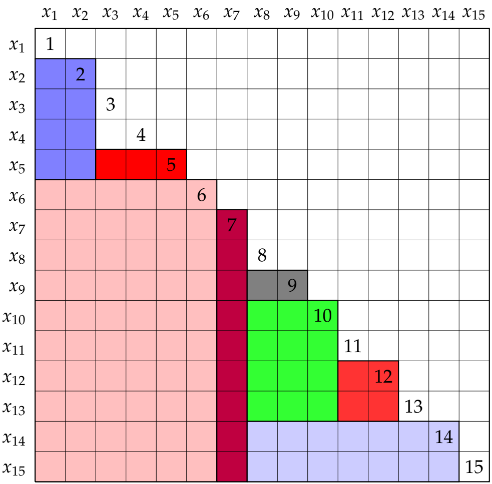

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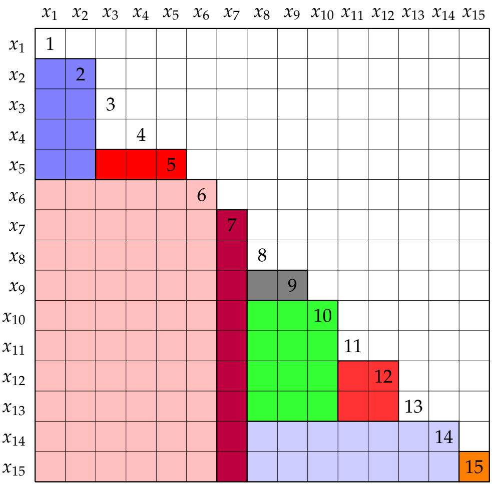
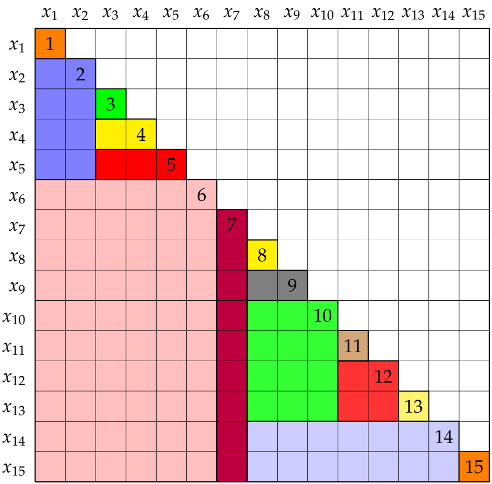


Example : A member of the adjacent rules family

$6 \rhd 7 \rhd 14 \rhd 2 \rhd 5 \rhd 10 \rhd 12 \rhd 9 \rhd 15 \rhd 1 \rhd 4 \rhd 3 \rhd 8 \rhd 11 \rhd 13$



A Geometric Characterization of the Adjacent Rules

Theorem 2

Consider a mediation rule f satisfying parts (i) and (ii) of Theorem 1. The following statements are equivalent:

(i) f satisfies part (iii) of Theorem 1.

(ii) $\triangle_{m,1}$ has a rectangular partition such that f assigns a unique bundle to each rectangle in this partition.

(iii) Precedence order $\triangleright$ is such that $f_{\ell,j} = \max_{\mathbf{B}_{\ell j}} \triangleright$

where $\mathbf{B}_{\ell j}$ is the set of mutually acceptable logrolling bundles at $(\theta_1^{\ell}, \theta_2^{j})$.

Logrolling

- Λ satisfies ***logrolling*** if $\exists$ a function $t : X \to Y$ such that $\forall\ i,\ \theta_i \in \Theta_i$, $R_i \in \Lambda(\theta_i)$ and all $x, x' \in A(\theta_i)$ with $x\ \theta_i\ x'$, we have

$$\left(x', t(x')\right)\ R_i\ \left(x, t(x)\right)$$

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$$(x', t(x'))\ R_i\ (x, t(x))$$

- Namely
  1. For any two acceptable x, x' in X where x is ranked above x', there are two y, y' in Y s.t (x', y') is ranked at least as high as (x, y) at all admissible R_i
  2. types are "consistent": order reversing mapping, t, is independent of types

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  2. types are "consistent": order reversing mapping, t, is independent of types

- Rules out Lexicographic preferences
- Quasi-linear preferences satisfy
- Cobb-Douglas preferences satisfy weak logrolling
- *Weak logrolling*: Sufficient and Necessary for possibility

Example (logrolling)

- $X = \{x_1, x_2, x_3, o_X\}$ and $Y = \{y_1, y_2, y_3, o_Y\}$

1	2
x_1	x_3
x_2	x_2
x_3	x_1

1	2
y_1	y_3
y_2	y_2
y_1	y_1

- Therefore, $\mathbf{B} = \{(x_1, y_3), (x_2, y_2), (x_3, y_2)\}$ set of logrolling bundles

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- $X = \{x_1, x_2, x_3, o_X\}$ and $Y = \{y_1, y_2, y_3, o_Y\}$

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- Therefore, $\mathbf{B} = \{(x_1, y_3), (x_2, y_2), (x_3, y_2)\}$ set of logrolling bundles
- Logrolling implies

$$(x_3, y_1)\ R_1\ (x_2, y_2)\ R_1\ (x_1, y_3)$$

and

$$(x_1, y_3)\ R_2\ (x_2, y_2)\ R_2\ (x_3, y_1)$$

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1	2
y_1	y_3
y_2	y_2
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- Therefore, $\mathbf{B} = \{(x_1, y_3), (x_2, y_2), (x_3, y_2)\}$ set of logrolling bundles
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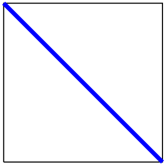
and

$$(x_1, y_3)\ R_2\ (x_2, y_2)\ R_2\ (x_3, y_1)$$

whenever all three alternatives in X are acceptable

- However, it only requires $(x_2, y_2)\ R_1\ (x_1, y_3)$ if x_3 is unacceptable for 1

The Possibility Result: Full Characterization



Theorem 2

Suppose prferences satisfy logrolling (quid pro quo). The mediation rule f is efficient, IR and SP if and only if the following hold:

(i) *If $\ell < j$, then $f_{\ell,j} = (o_X, y)$ for some $y \in Y$*

(ii) *If $\ell = j$, then $f_{\ell,j} = (x_\ell, \hat{y}_\ell)$ where $\hat{y}_\ell = y_{m+1-\ell} \in Y$*

(iii) *(Adjacency) If $\ell > j$, then $f_{\ell,j} \in \{f_{\ell-1,j}, f_{\ell,j+1}\} \subset \mathbf{B} = \{(x_\ell, \hat{y}_\ell) | \ell = 1, ..., m\}$ and there exists strict precedence order $\rhd$ on $\mathbf{B}$ such that*

$$f_{\ell,j} = \begin{cases} f_{\ell-1,j}, & \text{if } f_{\ell-1,j} \rhd f_{\ell,j+1} \\ f_{\ell,j+1}, & \text{oth.} \end{cases}$$

Special members of the adjacent rules family

Negotiator 1-optimal rule

- Recall that

$$R_1 : (x_5, y_1) \; R_1 \; (x_4, y_2) \; R_1 \; (x_3, y_3) \; R_1 \; (x_2, y_4) \; R_1 \; (x_1, y_5)$$

Negotiator 1-optimal rule

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- Negotiator 1-optimal rule takes $\triangleright \; = \; P_1$

	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_4}$	$\theta_2^{x_5}$
$\theta_1^{x_1}$	(x_1, y_5)	(o_X, y)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_2}$	(x_2, y_4)	(x_2, y_4)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_3}$	(x_3, y_3)	(x_3, y_3)	(x_3, y_3)	(o_X, y)	(o_X, y)
$\theta_1^{x_4}$	(x_4, y_2)	(x_4, y_2)	(x_4, y_2)	(x_4, y_2)	(o_X, y)
$\theta_1^{x_5}$	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)

Negotiator 2-optimal rule

- Recall that

$$R_2 : (x_1, y_5) \; R_2 \; (x_2, y_4) \; R_2 \; (x_3, y_3) \; R_2 \; (x_4, y_2) \; R_2 \; (x_5, y_1)$$

Negotiator 2-optimal rule

- Recall that

$$R_2 : (x_1, y_5) \; R_2 \; (x_2, y_4) \; R_2 \; (x_3, y_3) \; R_2 \; (x_4, y_2) \; R_2 \; (x_5, y_1)$$

- Negotiator 2-optimal rule takes $\triangleright \; = \; P_2$

	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_4}$	$\theta_2^{x_5}$
$\theta_1^{x_1}$	(x_1, y_5)	(o_X, y)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_2}$	(x_1, y_5)	(x_2, y_4)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_3}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(o_X, y)	(o_X, y)
$\theta_1^{x_4}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(x_4, y_2)	(o_X, y)
$\theta_1^{x_5}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(x_4, y_2)	(x_5, y_1)

Negotiator 1-optimal vs. Negotiator 2-optimal rule

- Negotiator 1-optimal rule:

	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_4}$	$\theta_2^{x_5}$
$\theta_1^{x_1}$	(x_1, y_5)	(o_X, y)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_2}$	(x_2, y_4)	(x_2, y_4)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_3}$	(x_3, y_3)	(x_3, y_3)	(x_3, y_3)	(o_X, y)	(o_X, y)
$\theta_1^{x_4}$	(x_4, y_2)	(x_4, y_2)	(x_4, y_2)	(x_4, y_2)	(o_X, y)
$\theta_1^{x_5}$	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)	(x_5, y_1)

- Negotiator 2-optimal rule:

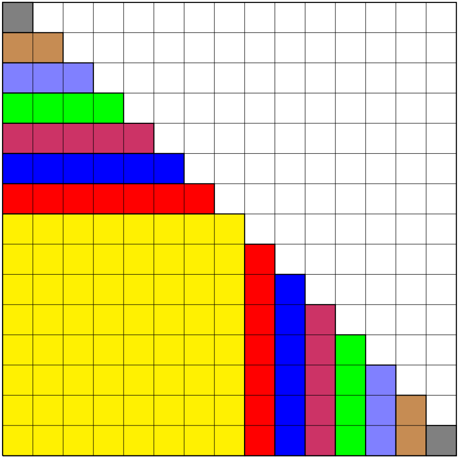
	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_4}$	$\theta_2^{x_5}$
$\theta_1^{x_1}$	(x_1, y_5)	(o_X, y)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_2}$	(x_1, y_5)	(x_2, y_4)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_3}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(o_X, y)	(o_X, y)
$\theta_1^{x_4}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(x_4, y_2)	(o_X, y)
$\theta_1^{x_5}$	(x_1, y_5)	(x_2, y_4)	(x_3, y_3)	(x_4, y_2)	(x_5, y_1)

Constrained Shortlisting Rule

- A Constrained Shortlisting rule rule takes $\rhd$: (x_3, y_3) (x_2, y_4) (x_1, y_5) and $\rhd$: (x_3, y_3) (x_4, y_2) (x_5, y_1)

	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_3}$	$\theta_2^{x_4}$	$\theta_2^{x_5}$
$\theta_1^{x_1}$	(x_1, y_5)	(o_X, y)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_2}$	(x_2, y_4)	(x_2, y_4)	(o_X, y)	(o_X, y)	(o_X, y)
$\theta_1^{x_3}$	(x_3, y_3)	(x_3, y_3)	(x_3, y_3)	(o_X, y)	(o_X, y)
$\theta_1^{x_4}$	(x_3, y_3)	(x_3, y_3)	(x_3, y_3)	(x_4, y_2)	(o_X, y)
$\theta_1^{x_5}$	(x_3, y_3)	(x_3, y_3)	(x_3, y_3)	(x_4, y_2)	(x_5, y_1)

Symmetry of the "Constrained Shortlisting Rule"



Constrained Shortlisting Rule: A Characterization

Constrained Shortlisting Rule: A Characterization

- Not only symmetric/impartial but also minimizes (within SP, efficient and IR rules) rank variance
- $r_i(z)$ is negotiator i's ranking of alternative $z \in Z$
- Given a mediation rule $f = [f_{\ell,j}]_{(\ell,j)\in M^2} = [(f^X_{\ell j}, f^Y_{\ell j})]_{(\ell,j)\in M^2}$
- Rank variance of the bundle $f_{\ell,j}$ is

$$var(f_{\ell,j}) = \sum_{i=1,2} \left(r_i(f^X_{\ell j})\right)^2 + \left(r_i(f^Y_{\ell j})\right)^2$$

- For any IR, efficient and SP rule f, **rank variance of f** is

$$Var(f) = \sum_{\ell=1}^{m} \sum_{j=1}^{m} var(f_{\ell,j})$$

Constrained Shortlisting Rule: A Characterization

Theorem 3

A mediation rule minimizes the rank variance within the class of efficient, individually rational and strategy-proof rules if and only if it is a Constrained Shortlisting rule.

Conclusion

- A weaker version of logrolling is both sufficient and necessary for a positive result: A strategy-proof, efficienct and individually rational rule exists iff the set of logrolling bundles form a semilattice. (SEE PAPER)
- Main model can be extended to the continuous case. (SEE PAPER)
- Logrolling (quid pro quo) is essential for strategy-proofness
- Argued that mediation can be made more rigorous and structured while maintaining practical simplicity
- Model can accommodate both transferable and nontransferable utility environments